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Number Concepts of Small Children

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In order to study the development of number concepts an investigation was made of those concepts held by two groups of children attending the Laboratory School of Southern Illinois Normal University. The first group were studied during the year of 1944–1945 and the second in the summer of 1945. The children were interviewed separately that the remarks of one might not influence the response of another. The teacher sat opposite each child at a small table on which had been placed a working sheet, fifteen small colored cubes, a toy truck containing 28 cubes, a pencil and a paper.

The same order was followed in each interview: (1) Counting by rote; (2) Counting objects; (3) Repeating a series of 4 numbers; (4) Picking out of a pile of 15 cubes a designated number; (5) Repeating a series of 5 numbers; (6) Adding two simple problems; (7) Subtracting one problem; (8) Recognizing the numbers 1 to 10; (9) Writing the numbers 1 to 10; (10) Responding to the questions "What do you think of when I say one?" this same question was asked for the first 10 numbers; (11) At the close of each interview the teacher recorded such remarks as she felt were significant.

Tables 1 and 2 contain the responses given to the questions; (1) counting by rote, (2) counting objects, (3) repeating 4 numbers, (6) repeating 5 numbers. Expressing their ages in months the children were arranged according to chronological age. Of the 44 children included in this study 23 entered the First Grade in the fall of 1945.

Not one of the 44 children used his fingers while counting by rote; yet it would be unwise to say that these children had already passed through the primitive man's stage of counting. Grimm (3) and Pott (4) maintain that primitive man used his fingers in his early stage of learning to count although Conant says, "In nearly all languages the origin of the words for 1, 2, 3, and 4 are so entirely unknown that speculation representing them is almost useless." Rather let us agree that these children have had excellent training in their homes.

Although the child might be able to count to 20, yet if he persistantly omitted 15, the number 14 is recorded on the work sheet, as that was as far as he was able to go without making a mistake. It will be noted that of the 23 children who entered the First Grade in the fall 96 per cent can count by rote beyond 10, 82 per cent to 15, 50 per cent beyond 20 and 22 per cent to 100. This is in close agreement with the findings of Buckingham and MacLatchy who found, from interviewing 1,290 children entering First Grade at the age of six,

TABLE 1

A Study of Counting and Repeating Numbers

Pupil	Age	Counting		Repeating Numbers					
		Rote	Object	4729	3852	7261	31859	48372	96183
S.S.	70	100	28						
L.B.	68	12	29				3189	3872	
D.L.	67	100	28						
L.C.	66	20	26						
L.F.	65	19	28				3859		
M.R.	65	100	28						
P.W.	65	23	25						96813
J.O.	64	13	25				3589	4372	961
F.H.	64	30	28						9683
M.H.	62	29	29					4872	978
S.H.	62	17	19					4872	9613
V.L.	62	100	28						
F.S.	61	15	28				3189	43782	
J.V.	60	14	29				31859	4315	
A.L.	59	18	23		3859				16983
M.G.	58	11	29						
W.P.	54	14	19						
J.C.	52	10	10		3859			4372	9763
T.W.	52	4	4	729	3899		1389	372	613
W.M.	51	10		729			3859	4	9683
C.T.	51	3					59	372	6183
S.T.	51	2	2				3859	4872	

TABLE 2

A Study of Counting and Repeating Numbers

Pupil	Age	Counting		Repeating Numbers					
		Rote	Object	4729	3852	7261	31859	48372	96183
B.O.	73	19	19			7621	3859	47	
C.S.	73	12	15			72461	3159	3672	9683
H.B.	72	100	4						
D.H.	72	20	28						
F.C.	71	14	14				1859	8172	
D.L.	71	39	28						
M.H.	70	13	16					4	96813
F.H.	68	14	14				1859	18372	781
R.H.	67	15	28				3859	4372	9683
C.R.	66	9	14	479	3892		13859	482	183
J.V.	64	15	15		3892		1356	4356	6452
C.H.	63	6	6		3894	7268	3859	3246	39139
M.P.	62	40	28						
W.P.	58	40	19						
J.R.	56	16	16				3859	4872	1683
J.C.	56	12	3						
S.T.	52	3	4					4782	
J.G.	42	Did not seem to be interested							
J.M.	36								
L.D.	34								
A.K.	34								

that 90 per cent could count to 10, 75 per cent to 15, and 60 per cent to 20. (1) Referring again to the tables it will be seen that two-thirds of the 4 year olds were counting to 10. While the two and three year olds did not respond at the time of the study little J.M. while resting one day repeated again and again in a sleepy sing song fashion, "One, two, three, four hup; one, two, three, four hup." It is evident that the concept here was that of her daddy marching.

Thus far it is evident that the children in this study are average in learning to

repeat number names, so we turn now to our basic question do they have a concept for these number names. In the study of object counting the two tables will be considered separately inasmuch as the children represented in Table 1 had attended kindergarten or nursery for 8 months before the study was made while those in Table 2 had only been given a week. The little toy truck was filled with cubes arranged 7×4 thus making 28 in all. According to Table 2 4 of the 13 five year olds counted blocks beyond their individual record in rote counting. According to Table 1 there were 8 of the 14, more than half, who counted beyond their rote counting. This would indicate that when the need for a number presented itself the child was challenged to produce one. Of the 12 who thus produced extra names all but one (C.R.) did so in either the teens or twenties at places where they might continue the pattern of number naming—fourteen, fifteen, sixteen. In the case of C.R. the names for ten, eleven and twelve were necessary and it is more than likely that it was the process of recall that produced these names, but it was the challenge of the situation which brought about this recall. The fact that almost half of these five year olds are counting objects beyond their rote counting indicates the fact that rote counting is not a necessary prelude to object counting. Moreover rote counting proved to be a handicap to H.B. who glibly rattled off the number names to 100 but was caught at 4 in counting objects. Upon being asked to count the number of children in attendance his blank expression told us that he was definitely lost. Yet upon listening to another count the children a pleased expression of recognition passed over his face which seemed to say, "So that is the way you use those words."

That children can easily learn the number names or at least can reproduce them upon hearing them repeated is demonstrated by the facts presented in columns 5 through 10 of Tables 1 and 2. Columns 5, 6, and 7 contain the numbers found in the 6th month of the 4th year, Form "L" of the Revised Stanford-Binet. All of the 5 year olds recorded in Table 1 and 8 of the 13 of Table 2 or a total of 82 per cent reported all three sets of numbers correctly and 83 per cent of the 4 year olds repeated them correctly. It will be noted that all but one of the 5 year olds as shown in Table 1 got at least 1 of the 5 numbers correct. Considering the responses of the 2 tables together we find that 33 per cent could repeat the sets of 5 numbers correctly, while the most of them could remember 4 of the 5 numbers and place them in their correct order. Thus we may conclude that children can readily master number names and reproduce them in any sequence and that the more the children work with number names the easier it is for them to reproduce them.

Having seen how these children count by rote, count objects and reproduce number names in a specified order let us turn to the findings presented in Tables 3 and 4. Columns 3, 4, 5, and 6 contain the responses made by the children upon being called on to pick out of a pile of 15 blocks 3, 9, 5 and 7. This problem is taken from the 6th year of Form "L" Revised Stanford-Binet. By setting directly opposite the child the teacher was able to observe the child's attack on his problem. The top line of each child's responses records the number of blocks he picked up at a time; when it was the "one by one" attack the three ones were recorded; where the child picked up more than one at a time the number is recorded in the order in which they were picked up. The second line contains the symbol indicating the verbal or silent reaction while attacking the problem—C, counting. CA counting aloud, CE, counting with the eyes. Sitting opposite the child, the teacher was in a position where she might observe the eye fixations. Where the response was correct no number was recorded; where there was an error this was recorded. Of the children who had attended kindergarten-nursery

TABLE 3

A Study of Number Concepts

Pupil	Age	Concept				Addition		Subtraction
		3	9	5	7	3+2	4+2	5−2
S.S.	70	111 CE	111 CE	111 CE	111 CE	CE	CE	CE
L.B.	68	111 CE	111 CA	111 CA	111 CA	CA	CA	CE
D.L.	67	111 CE	111 CA	111 CA	111 CA	CE	CE	CE
L.C.	66	111 CE	111 CA	111 CA	111 CA	CE	4,6 CA	CE
L.F.	65	111 CE	111 CE	111 CE	111 CE	CE	CA	CE
M.R.	65	111 CE	22221 CE	221 CE	2221 CE	CE	CE	CE
P.W.	65	111 CA	111 CA	111 CA	111 CA	CA	CE	CE
J.O.	64	12 CA	111 CA	111 CA	111 CA	CA	CA	CA
F.H.	64	111 CA	111 CA	111 CA	111 CA	CE	CE	CE
M.H.	62	21 CA	111 CA	111 CA	111 CA	CA	CA	CA
S.H.	62	111 CA	311 CA	111 CA	111 CA	CA	CA	CA
V.L.	62	111 CE	111 CE	111 CE	111 CE	CA	CE	CE
F.S.	61	3 CE	333 CE	41 CE	331 CE	CE	CE	CE
J.V.	60	21 CE	111 CA	111 CA	1222 CE	CE	CE	CA
A.L.	59	111 CA	111 CA	111 CA	111 CA	CE	CA	CA
M.G.	58	111 CE	3111 CE	111 CE	3111 CE	CE	CA	CE
W.P.	54	111 CE	3111 CA	3111 CA	3111 CA	4,5 CA	CA	CA
J.C.	52	111 CA	111 CA	111 CA	111 CA	CA	CA	CA
T.W.	52	111 CA	111 6CE	111 4CE	111 6CE	CA	"Dont Know"	"Dont Know"
W.M.	51	111 CA	111 CA	111 CA	111 CA	CA	CA	CA
C.T.	51	111 CA	111 CA	111 CA	111 CA	CA	CA	CA
S.T.	51	11 2CA	11 2AC	11 2CA	11 2CA	2CA	2CA	2CA

CA, counting out loud; CE, counting with the eyes.

for 8 months there were 21 of the 22 or 95 per cent who picked up the correct number of blocks each time; of the 4 and 5 year olds who had attended only a week there were 14 of the 17 or 82 per cent. Combining the two groups we find that 92 per cent of the 4 and 5 year old children have developed number concepts up to 10.

But how are they building these concepts; how do they pick up these blocks and do they talk to themselves while they are doing it? Upon first glance it would appear that 4 and 5 year old children are still in the "one by one" stage of development for of the 156 problems 135 or 86 per cent were attacked by the children in the "one by one" pattern. From the pile of 15 blocks they pulled or lifted out their

TABLE 4

A Study of Number Concepts

Pupil	Age	Concept				Addition		Subtraction
		3	9	5	7	3+2	4+2	5−2
B.C.	73	111 CE 111	111 CA 111	111 CE 111	111 CA 111	CE	CE	CE
C.S.	73	CE 111	CE 111	CA 111	CA 111	CE	CE	CE
H.B.	72	CA 111	4CA 111	CA 111	4CA 111	D−K	D−K	D−K
D.H.	72	CE 111	CA 111	CE 111	CA 111	CE	CE	CE
E.C.	71	CE 111	4CA 111	CA 111	CA 111	CE	CE	CE
D.L.	71	CA 111	CA 111	CA 111	CA 111	CA	CA	CA
M.H.	70	CA 111	CA 111	CA 111	CA 111	CA	CA	CA
F.H.	68	CE 111	CA 111	CA 111	CA 111	CE	CE	CE
R.H.	67	CE 111	CE 111	CE 111	CE 111	CE	CE	CE
C.R.	66	CE 111	CA 111	CA 111	CA 111	CE	CA	CE
J.V.	64	CA 111	CA 111	CA 111	CA 111	CA	CA	CA
C.H.	61	CE 111	CE 111	CE 111	CE 111	2CE	10CE	CE
M.P.	62	CE 21	CA 22221	CE 221	CA 2221	CE	CE	CE
W.P.	58	CE 111	CE 111	CE 111	CE 111	CA	CA	CE
J.R.	56	CA 111	CA 111	CA 111	CA 111	CA	CA	CA
J.C.	56	CA 111	CA 111	CA 111	CA 111	CA	CA	CA
S.T.	55	CA	3CA	3AC	4CA	2CA	2CA	2CA
J.G.	42	Did not seem interested						
J.M.	36							
L.D.	34							
A.H.	34							

CA, counted aloud; CE, counted with eyes; D−K "Dont Know."

blocks one by one bringing them nearer to themselves thus making a second pile of those they had selected. According to Table 3 there were 7 of the 22 or 31 per cent who used a 2 by 2 or a 3 by 3 pattern of approach; only 1 of the second group used a 2 by 2 pattern of approach. Of these 8 children 4 were girls and 4 boys so the sex does not seem to be the key to the superior approach; neither does age, for of these 8 children 3 were 4 year olds; rather it was found that those children who used the 2 by 2 or the 3 by 3 approach were those who had been found most often playing with blocks, thus it may be experience.

Upon studying our second question—do 4 and 5 year olds talk while solving their mental problems—we find that 34 per cent of the problems were solved silently. Considering this from the point of view of age we find that 75 per cent of the 6 year olds, 60 per cent of the 5 year olds and 40 per cent of the 4 year olds solved their first problem silently. Had we known the mental maturity it would doubtless have proved more interesting. Considering it now from the point of view of the individual problems, 56 per cent picked up the 3 blocks silently, 23 per cent picked up the 5 silently, 25 per cent picked up the 7 silently while 28 per cent picked up

the 9 silently. Arranging these as they were solved we have 56 per cent for the 3 28 per cent for the 9, 23 per cent for the 5 and 25 per cent for the 7. This would seem to indicate that two factors entered into the breaking from the silent attack started in the solving of the first problem: (1) difficulty, and (2) fatigue, that is weariness resulting from working on the previous problems. We may conclude that: (1) at least 90 per cent of 4 and 5 year olds have developed concepts for the first 10 ordinals; (2) kindergarten-nursery experience facilitate this development; (3) 86 per cent of the children this age still attack their problems in a "one by one" pattern of behavior—experience plays a dominant factor in this development; (4) while age is a dominant factor in the child shifting from the oral to the silent approach, the ease of the problem to be solved and fatigue play important roles.

In order to see how these children combine two numbers to get a third or subtract one number from another these problems were included. It will be noted that two of the 3 number concepts (3 and 5) have been used in this study before. Introducing the first problem the teacher, acting as she spoke, said, " . . . I am giving you 3 blocks, now I am giving you 2 more blocks. Tell me how many do you have?" For the second problem, "This time I am giving you 4 blocks, now I am giving you 2 more blocks. Tell me how many blocks do you have?" In the case of the third problem, "See I have given you 5 blocks now I am taking 2 away. Tell me how many do you have?" Each time the teacher was careful to arrange the blocks according to the patterns shown at the foot of Table 3. In order to answer the question the child might either continue counting from the number first laid down or count the total number. In each case where he spoke aloud he was heard grouping the number names 1, 2, 3–4, 5 and 1, 2, 3, 4–5, 6. In the case of the two children who replied "Don't know" it was evident that they saw in these problems something too big to attempt for their behavior pattern said, "I don't know and

TABLE 5

Study of the Ability to Recognize and Reproduce Figures 1 to 10

Pupil	Recognize										Reproduce									
	3	9	5	7	1	2	4	6	8	10	1	2	3	4	5	6	7	8	9	10
S.S.	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
L.B.	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x				
D.L.	x	x	x	x	x	x	x	x	x	x	x	r	r	r	x	x	x	x		
L.C.	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	
L.F.	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x					
M.R.	x	x	x	x	x	x	x	x	x	x	x	r	r	r	x	x	r			
P.W.	x	x	x	x	x	x	x	x	x	x	x	r	r							
J.O.	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x					
F.H.	x	x	x	x	x	x	x	x	x	x	x	r	x	x	x	x	x	x	r	x
M.H.	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
S.H.	x				x	x					x	r	x							
V.L.	x	x	x	x	x	x	x	x	x	x	x	r	x	x	x					
F.S.	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x					
J.V.	x				x	x					x									
A.L.																				
M.G.	x		x	x	x	x	x	x	x	x	x	r	r	x	x	x	r	x		
W.P.					x	x														
J.C.																				
T.W.																				
W.M.					x	x														
C.T.																				
S.T.																				

x, the number was recognized, written. r, number reversed.

TABLE 6

Study of the Ability to Recognize and Reproduce Figures 1 to 10

Pupil	Recognize										Reproduce									
	3	9	5	7	1	2	4	6	8	10	1	2	3	4	5	6	7	8	9	10
B.C.	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x		x			
C.S.			x		x	x	x	x			x	x		x		x				
H.B.	x		x	x	x	x	x	x			x	x	x	x	x	x	x			
D.H.	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
F.C.	x	x	x	x	x	x	x	x	x	x	x	r	x	x	x	x	r	x	r	x
D.L.	x	x	x	x	x	x	x	x	x	x	x	x	x							
M.H.	x				x	x	x				x	x		x						
F.H.	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	r	x	x	x
R.H.	x		x		x	x	x				x	x	x	x	x	x	x			
C.R.					x						x									
J.V.					x	x					x	x								
C.H.																				
M.P.	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
W.P.	x				x	x					x									
J.R.																				
J.C.																				
S.T.																				
J.G.																				
J.M.																				
L.D.																				
A.K.																				

I am making no attempt to find out." One of these was H.B. who could glibly pronounce number names to 100 but made a mistake on 5 when counting objects. However there were only 2 of the 39 who thus responded. In other words 94 per cent took the solution of these problems in their stride and 96 per cent of these responses were correct—yet there are those who would place these simple problems in the second grade.

Of these 39 children 46 per cent solved the first problem silently, 41 per cent the second while 53 per cent solved the problem in subtraction silently. Comparing these with the previous study we find that 56 per cent picked 3 blocks out of the 15 silently and now there were 53 per cent who solved the subtraction problem silently. We have 23 per cent picking up 5 blocks silently but 46 per cent, almost double, adding the 3 plus 2 silently. It would seem that the idea of adding two more to the 3 that were already down was easier than the picking of the total number of 5 out of a larger number. The same holds for the second problem of addition for only 25 per cent pulled out 7 blocks silently while 41 per cent added 2 to the 4 blocks silently. It may therefore be con-

TABLE 7

The Percentages of the Children Who Were Able to Recognize and Reproduce the Numbers 3, 5 and 10

Group	Numbers		
	3	5	10
Recognize			
1	100%	90%	90%
2	76%	53%	35%
Total	89%	72%	66%
Reproduce			
1	95%	86%	13%
2	47%	41%	23%
Total	72%	64%	20%

TABLE 8

Number Concepts

Number	Limited for all		Limited for some		Possible answers	
1	Bead Mouth Nose Neck Tongue Stomach	Attic Mother Father Ice box Sewing Machine	Brother Sister Bathroom Kitchen Dining Room	Dog Cat Car Maid Truck Toy wagon	Chair Block Pencil Table Tree Engine	
2	Arms Ears Thumbs Jaws Elbows Feet	Shoes Cheek G. mother G. father Legs	Brothers Sisters Bedroom Bathrooms Cats Dogs	Cars Maids Dolls Guns Porches	Blocks Chairs Windows Hammers Houses	
3			Sisters		Paste jars Chairs Blocks Papers	Books Nails Dolls
4	Legs on chair table dog cat bed	Wheels on car truck wagon	Rooms Milk bottles		Trees Chairs Blocks Books Pencils Curtains	
5	Fingers Toes		Rooms Milk bottles		Hanners Blocks Books Crayons	Papers Chairs Houses
6			Apples Crackers Plums Cups Crayons	Napkins Doilies Pencils Papers Spoons	Trees Houses Cars Trucks People	
7	Days in a week		Rooms at home		Rooms Trees Blocks Books	Beads
8			Rooms at home		Stones Blocks Beads	Books Pencils
9			Rooms at home		Hairs Whiskers Trees	Cars Pencils
10	Fingers Toes		Rooms at home		Chairs Pencils Girls Cups	Boys Books Nails Beads

cluded that 4 and 5 year old children add and subtract readily simple concrete problems.

To make sure that the response was that of recognition rather than the recall of a number name in a series, the numbers —in the order in which they occur in Table 5 and 6—were written down one by one as the child responded. The findings are presented in Tables 5 and 6 while the percentages for numbers 3, 5 and 10 may be found in Table 7. Following the opportunity to recognize numbers the child was handed a pencil and a piece of paper and asked to write down as many numbers as he could. These findings are also in-

cluded in Tables 5 and 6 and the percentages for numbers 3, 5 and 10 are to be found in Table 7. From these findings we may conclude that: (1) 72 per cent of 5 year old children can recognize the first 5 numbers and 64 per cent of them can reproduce them without help; (2) 66 per cent of 5 year old children can recognize numbers 1 to 10 and 20 per cent can without help reproduce them; and (3) children attending kindergarten are further advanced than those not having this experience.

The responses on the 11th item, "Tell me what do you think of when I say 1, . . . when I say 10," are to be found in Table 5. It will be noted that these are grouped under (1) Limited for all, (2) Limited for some, and (3) Possible answers.

The responses in Table 5 substantiate the following conclusions:

(1) Children 4 and 5 years old have very clear and definite concepts for the numbers: 1, 2, 4, 5 and 10. The kindergarten-nursery children of Southern Illinois Normal University have a clear concept for 6 besides 1, 2, 4, 5 and 10 inasmuch as they happen to have 6 seated at each table.

(2) The number of the various parts of the child's own body aid in building the number concepts.

As a postlude let us enter the homes of these children for a week and see that arithmetic concepts and problems attract their attention. The mothers are keeping a record and the responses are to be found in Tables 9 and 10. From these responses we find that: (1) Children 4 and 5 years of age have a wide range of arithmetic activities, (2) Children 3 years of age are using such arithmetic concepts as time, money and length besides that of counting, adding and subtracting. The reader will be interested in a few of the incidents recorded by these mothers.

TABLE 9

Home Interests in Arithmetic

Age	Counting		Numbers		Measuring		
	Rote	Object	Reading	Writing	Length	Weight	Liquid
5	x	x	x	x	x	x	x
4	x	x	x	x	x	x	x
3	x	x			x		
2	x	x					

TABLE 10

Home Interests in Arithmetic

Age	Time	Money	Addition	Multiplication	Subtraction	Division	Fractions
5	x	x	x	x	x	x	x
4	x	x	x	x	x	x	x
3	x	x	x		x		
2		x					

FIVE YEAR OLDS

1. "We were hunting 616 N. Almond and S called off house numbers and finally located 616."

2. "Measured her new puppy with yard stick."

3. "Weighed herself on the bathroom scales."

4. "Poured a quart of water in the milk bottle to see if they were the same."

5. "Knows the days of the week, Saturday—dancing school; Sunday—Sunday school.

6. "Knows 5:15 P.M. definitely—'Superman time.' "

7. "Ate chocolate drops from the pudding subtracting as she ate."

8. "Cary got $15.00 for 15 chickens and I got $2.00 for 10 chickens. It isn't fair."

9. "Has read page numbers up to 435."

10. "Always knows the amount of money he has, checks his allowance."

11. "S uses arithmetic daily, for example he returned from store with one stick of butter and said, 'That was all the the change you gave me. If I had gotten 3 more sticks I'd have a pound.' "

12. "Can divide food or toys equally among the children he plays with."

13. "Adds by using pennies, beginning with 1+1 =2 through combinations totaling 6."

Four Year Olds

1. "Conception of time: daylight, night time, meal time."

2. "Uses rote system of counting money, not its value."

3. "Knows 5 pennies make a nickle and 2 nickles make a dime."

4. "Knows penny, nickle, dime and quarter."

Three Year Olds

1. "J had $1.00 to buy a bathing suit. I asked her if she wanted one of the $2.98 ones. She said no that was too much money. (She understood from a Christmas deal when she bought a doll buggy without having enough money to pay for all of it. Then she had to pay for it as she got the money. Their money usually comes from relatives who give them from 25¢ to a $1.00 each time they come. They get 10¢ a week allowance.) J. found a suit at Woolworths for 32¢ (a rather nice one for her too). Then she bought a flag for 20¢, some candy for 10¢, the rest she has spent a penny at a time."

2. "Counts out red points."

3. "Knows 12 dinner; 6 o'clock, supper; 7:30 bed time."

4. "Know that when the Panama passes through it is time for bed."

Summary

In order to study the development of number concepts an investigation was made of those held by 4 and 5 year old children attending kindergarten-nursery at Southern Illinois Normal University. Seated at a small table containing the work sheet, 15 blocks, a toy truck containing 28 cubes, a pencil and a paper the teacher interviewed the 44 children separately that the remarks of one might not influence the response of another. The same order was followed in each interview: (1) Counting by rote; (2) counting objects; (3) repeating a series of 4 numbers; (4) picking out of a pile of 15 blocks a designated number; (5) repeating a series of 5 numbers; (6) adding 2 simple problems; (7) subtracting one problem; (8) recognizing the numbers 1 to 10; (9) writing the numbers 1 to 10; (10) responding to the questions, "What do you think of when I say 1, . . . when I say 10." To the responses thus obtained are added the findings turned in by the parents of these same children.

The findings presented in the body of this manuscript may be summarized:

(1) Of the children who will be entering school in the fall 90 per cent can count by rote to 10, 82 per cent to 15, 50 per cent to 20 and 22 per cent to 100.

(2) Children of this age can readily master number names and reproduce them in any sequence. The more they work with numbers the easier it is to reproduce them.

(3) The fact that almost half the children this age are counting objects beyond their rote counting indicates the fact that rote counting is not a necessary prelude to object counting.

(4) Around 92 per cent of the 4 and 5

year old children have developed concepts for numbers 1 to 10, those most clear and definite are 1, 2, 4, 5 and 10. The number of the various parts of the body play a decided role in the building of these concepts.

(5) Around 72 per cent of the 4 and 5 year olds recognize the first 5 ordinals and 64 per cent can reproduce them while 66 per cent can recognize the first 10 ordinals and 20 per cent can reproduce them.

(6) While working and playing with concrete objects 4 and 5 year olds are adding and subtracting with surprising accuracy.

(7) Children 4 and 5 years of age are using arithmetical concepts both at home and at school. Those who have experience at both places have an advantage.

(8) Around 86 per cent of 4 and 5 year old children attack the problem of finding a number in the group in a "one by one" pattern. Experience aids in developing a two by two or a three by three pattern.

(9) Small children think out loud, yet as they grow older they increasingly use a silent approach—75 per cent for 6 year olds, 60 per cent for 5 and 40 per cent for 4 year olds.

BIBLIOGRAPHY

1. BUCKINGHAM, B. R., and MACLATCHY, J., "The Number Abilities of Children When They Enter Grade One," Twenty-ninth Year Book of National Society for the Study of Education, pp. 473–524.
2. CONANT, L. L., The Number Concept, New York, Macmillan and Co. 1896, p. 50.
3. GRIMM, Geschichte der deutschen Sprache, vol. 1, p. 239.
4. POTT, Zählmethode, pp. 29–45.

Eighty-first Regular Meeting of the Association of Mathematics Teachers of New Jersey

Atlantic City, 10:30 A.M., Hotel Claridge, December 1, 1945

MATHEMATICS IN A WORLD SOCIETY

10:30 A.M. Greetings—Dr. Fred L. Bedford, President
10:40 A.M. The Use of Multisensory Aids in the Classroom
Miss Madeline Messner, Roselle, N. J.
11:00 A.M. Experimental Development of Statistical Ideas
Professor Paul C. Clifford, Montclair, N. J.
11:20 A.M. Summary of the Post-War Commission's Study
Dr. Howard F. Fehr, Chairman; Montclair, N. J.
11:40 A.M. Teaching Arithmetic in Peace-Time
Clifford B. Upton, Professor Emeritus of Mathematics
Teachers College, Columbia University
12:30 P.M. Adjournment
2:30 P.M. Meeting of the Council.

EDUCATION TO PROMOTE THE GENERAL WELFARE
is the theme of the twenty-fifth annual observance of
American Education Week
which will be observed November 11–17, 1945.

Measurement of Success in College Mathematics

By I. L. STRIGHT
Baldwin-Wallace College, Berea, Ohio

HOW CAN one construct a valid examination in college mathematics? The test is to be completed in a one or two hour period. One problem may be given which, although representative of the work covered, may require the majority of the time available for testing. Does such a problem or a group of several problems, usually less than ten, give an accurate measure of how well a student has mastered the material covered?

It is possible to purchase certain objective tests for beginning courses in college mathematics, which the writer feels have limited value in use. These tests do not answer the needs raised above for other college courses or offer any help in the testing of units within courses.

The ideas and sample presented here is not intended as a panacea for measuring progress in college mathematics but is one attempt to arrive at a valid answer. For several terms the writer has been dividing his examinations into two parts, each part to be given in a one-hour period.

It is felt that a student should be able to use the tools and techniques of the particular unit or units so that he can analyze a problem and solve it in its entirety. Thus the conventional examination of a very few carefully selected problems are of indispensable value. One part or one hour of the two-hour examination is given to this type of problem.

The writer feels, however, that the solution of a few problems in itself is inadequate to give a valid measure of the student's achievement in the subject. The student's mastery of the terminology, acquaintance with the theory of many of the underlying principles, his ability to associate terms and algebraic expressions, cannot possibly be measured in detail in the solution of the number of problems workable within the limited time.

Therefore, the second part of the writer's tests are objective in nature and cover many details in the theory and processes within the subject material. The sample submitted here was used in a differential calculus class. This particular class had covered the usual material given in the first semester of a beginning calculus course. In addition to the work in the text in problem solving, these students were required to do some outside reading in some of the books written for laymen; such as Hogben's "Mathematics for the Millions" and Kokomoor's "Mathematics for Human Affairs."

This sample of the objective part of the examination contains four sections. The first section contains over thirty statements to be marked true or false; and tests terminology, definitions, and general knowledge of material. The second section is one of matching statements or phrases. An attempt is made here to measure the mastery of concepts in detail. The third section is one of matching formulas and attempts to measure the student's familiarity with important rules stated in algebraic symbolism. The fourth section, in the form of completion statements, attempts to measure the student's command of the basic processes, in this case the fundamental rules of differentiation.

A correction for pure guessing should unquestionably be applied to the true-false section; the score being the number right minus the number wrong. In the matching sections the probability of pure guessing is very slight. The score may safely be taken as the number right. If, however, it is felt that a correction should be made, there are several formulas which are used for this purpose. $S=R-(W/u-R)$[1] where "S" is the obtained score, "R" is the number right, "W" is the num-

[1] Chen, L., "The Correction Formula for Matching Tests." *Journal of Educational Psychology*, Dec., 1944.

Name_____

Part I

Plus—true; Zero—false

_____ 1. Calculus is an older branch of mathematics than geometry.
_____ 2. Most credit for the development of calculus is given to Newton and Leibniz.
_____ 3. The symbols used by Leibniz were superior to those of Newton.
_____ 4. Newton and Leibniz were very close friends.
_____ 5. The concept of a limit was a stumbling block in the development of mathematics for centuries.
_____ 6. Newton introduced the "dx" symbolism.
_____ 7. In the calculus, integration is sometimes called the "return trip."
_____ 8. Newton discovered the principle that successive swings of a pendulum occupy the same time.
_____ 9. Differential Calculus is a means of finding how steep a curve is at any point.
_____10. Problems of motion were in no way responsible for the development of calculus.
_____11. In real life most things move at a constant speed.
_____12. Time rates are derivatives with respect to time.
_____13. Differential and integral calculus employ methods which are in no way related.
_____14. The *one* inventor of calculus is known.
_____15. Leibniz was a student of Professor Barrow.
_____16. The integral calculus attained its greatest usefulness when coal as a source of energy supplanted human labor.
_____17. The derivative of a product of a constant and a function is equal to the derivative of the function.
_____18. The derivative of x with respect to y is equal to the reciprocal of the derivative of y with respect to x.
_____19. The differential of a function is an accurate value of the function.
_____20. The differential of a function equals the derivative of the function times an increase in the independent variable.
_____21. If an equation in x and y is solved for y, each of the resulting expressions is an implicit function of x.
_____22. In calculus the degree system for measuring angles is more practical than the radian system.
_____23. The radian is that central angle of any circle which is subtended by an arc of the circle equal to the radius.
_____24. The logarithm of a number is really an exponent of some number.
_____25. pi is a transcendental number.
_____26. Newton's method is helpful in finding rational roots.
_____27. The curvature of a circle is constant.
_____28. Evolutes are of no practical significance.
_____29. Certain physical quantities, such as velocity, can be represented by vectors.
_____30. The truth of Rolle's theorem is geometrically obvious.
_____31. Leibniz and Newton lived in the 15th century.

Number right _____
Minus number wrong _____
Score _____

Part II

_____ a. Newton's Method for solving equations.
_____ b. Parameter
_____ c. Curvature
_____ d. Evolute
_____ e. Logarithmetic differentiation
_____ f. Radian Measure
_____ g. Circle of curvature
_____ h. Time-rates
_____ i. Polar coordinates
_____ j. L'Hospitals Rule
_____ k. Points of inflection
_____ l. Symmetry with x-axis
_____ m. Critical points
_____ n. Asymptote
_____ o. Symmetry with y-axis
_____ p. Acceleration
_____ q. Transcendental number "e"

1. Method of differences
2. Mechanics of rest
3. Position designated by one distance and one direction
4. Force times distance
5. Irrational roots
6. No change when minus y replaces plus y
7. Linear measure of angle
8. Locus of centers of curvature
9. Limit of a curve
10. Method of indivisibilities
11. Break in curve
12. Small circle rolling around the inside of larger circle
13. Mechanics of motion
14. Radius equals the reciprocal of curvature of given point on a given curve.
15. Third variable, with a definite relation to two or more other variables.
16. Change in velocity
17. Indeterminate forms
18. Base of natural logarithms
19. Test for points of inflection
20. Position designated by two distances

_____ r. Method of Leibniz
_____ s. Statics
_____ t. Rolle's Theorem
_____ u. Method of Cavaleri
_____ v. Rectangular coordinates
_____ w. Dynamics
_____ x. Mean-value theorem
_____ y. Hypocycloid

21. Tangent parallel to a chord
22. Horizontal tangent somewhere between successive roots a and b
23. Change in direction of bending
24. Derivative of angle of inclination with respect to arc length
25. Derivative with respect to time
26. Technique for differentiating a function "u" to a power "v"
27. No change when minus x replaces plus x
28. Horizontal tangents

Part III

_____ a. ds
_____ b. K
_____ c. R
_____ d. X
_____ e. v
_____ f. a
_____ g. ω
_____ h. $\tan \psi$
_____ i. $D_x e^u$
_____ j. $D_x a^u$
_____ k. $D_x \log_a u$
_____ l. $D_x \text{ arc sec } u$
_____ m. $D_x \sin u$
_____ n. $D_x \text{ arc csc } u$
_____ o. $D_x \sec u$
_____ p. $D_x \cot u$
_____ q. e
_____ r. $\log_b N$
_____ s. $\log_a M^k$
_____ t. $D_x(uv)$
_____ u. $D_x\, cu$
_____ v. $D_x \dfrac{u}{v}$
_____ w. $D_x \text{ arc sin } u$
_____ x. $D_x \text{ arc tan } u$
_____ y. $D_x \cos u$
_____ z. $D_x \text{ arc cot } u$
_____ aa. $r \cos \theta$
_____ bb. r^2
_____ cc. $\log_e x$
_____ dd. θ
_____ ee. $r \sin \theta$

1. x^2+y^2
2. $\ln x$
3. $\dfrac{vD_xu - uD_xv}{v^2}$
4. $\dfrac{D_xu}{1+u^2}$
5. $k \log_a M$
6. $\dfrac{1}{K}$
7. $\left|\dfrac{d\alpha}{dx}\right|$
8. $\dfrac{d\theta}{dt}$
9. $e^u D_x u$
10. $\text{Cos } u\ D_x u$
11. $\lim\limits_{z\to 0} (1+z)^{1/z}$
12. $\dfrac{D_xu}{u}$
13. $\log_a N \log_b a$
14. x
15. $\dfrac{D_xu}{\sqrt{1-u^2}}$
16. cD_xu
17. $\sqrt{1+(y')^2dx}$
18. $\sqrt{\left(\dfrac{d^2x}{dt^2}\right)^2 + \left(\dfrac{d^2y}{dt^2}\right)^2}$
19. $x - \dfrac{y'[1+(y')^2]}{y''}$
20. $\sqrt{v_x^2+v_y^2}$
21. y
22. $\dfrac{-D_xu}{1+u^2}$
23. $y + \dfrac{1+(y')^2}{y''}$
24. $\text{arc tan } \dfrac{y}{x}$
25. $uD_xv + vD_xu$
26. $-\sin uD_xu$
27. $\dfrac{1}{u} \log_a e\ D_xu$
28. $-\csc^2 uD_xu$
29. $\dfrac{-D_xu}{\sqrt{1-u^2}}$
30. $\dfrac{D_xu}{u\sqrt{u^2-1}}$
31. $\dfrac{r}{D_\theta r}$
32. $a^u \ln a D_xu$
33. $\dfrac{-D_xu}{u\sqrt{u^2-1}}$
34. $\sec u \tan uD_xu$

Part IV

Write the derivatives of y with respect to x:

1. $y=(1+2x)^3$ _____
2. $y=\dfrac{2}{x^2-3}$ _____
3. $y=\sin 2x$ _____
4. $y=csc(4x-1)$ _____
5. $y=x \sin x$ _____
6. $y=\text{arc sin } (1-x)$ _____
7. $y=\ln ax$ _____
8. $y=e^{7x}$ _____
9. $y=\sin (\ln x)$ _____
10. $x=t^2,\ y=t^2$ _____
11. $y=x^3-x^2+x-1$ _____
12. $y=\ln(\sin x)$ _____
13. $y=\ln (x-2)^2$ _____
14. $y=\sec (4x+3)$ _____
15. $y=\sqrt{\sec 3x}$ _____

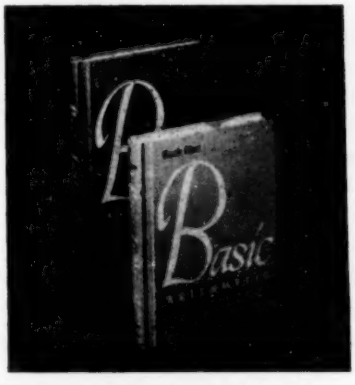
Score (2 pts. each) _____

ber wrong, and "u" is the number possible to match correctly, is one of the latest suggested for this purpose. The completion section in this sample has been scored with two points per blank. Thus all four sections have about equal value in determining the total score on the objective part.

This sample is not proposed as a perfect measuring stick but only as an attempt to provide an examination which gives the student an opportunity to show his mastery of the items tested. It is emphasized that this sample is not suggested as a complete test in itself but only as a supplement to the first part of the examination mentioned earlier. The two parts of the examination, problem-solving and objective, are given about equal weight in the student's final score.

The writer has been using this technique of construction of tests in several courses for some time. He has found it rather easy to construct the tests in reasonable time. Hectograph carbons of the objective parts can easily be made; either typed or in longhand. No effort has been made to use the multiple-choice technique of item construction as it did not seem to be satisfactory.

The reaction from many students has been extremely favorable, with the comment that the two parts in the total examination gives a much fairer opportunity for the student to show what he knows. With only a limited number of problems, regardless of how carefully they may be selected, the student may blame his low score on the particular selection.



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Redirection or Return to Direction in Mathematics

By LEHMAN HOEFLER
State Department of Education, Hartford, Conn.

FOR THE PAST eight years the Bureau of Youth Services of the Connecticut State Department of Education, which is charged among other responsibilities with the general supervision of instruction in the secondary schools in Connecticut, has been especially and deeply concerned about the outcomes of high school teaching in the state.

Under the direction of Commissioner Alonzo Grace, and Director Paul D. Collier, the Bureau engaged during 1939–1940–1941 in widespread evaluations of the general instruction in some ninety high schools in Connecticut, using the evaluative criteria and techniques of the Cooperative Study of Secondary School Standards. This statewide study was followed by the writing of several bulletins in specific areas, and finally by the presentation in December, 1944, of a coordinated program for secondary education which is intended to serve as a pattern, flexible but still fundamental, forward looking but still secure, for the future growth of Connecticut high schools. This bulletin, #37, "The Redirection, Reorganization and Retooling of Secondary Education," has certain direct implications for the field of mathematics, and makes certain proposals which are intended to help our instruction in the "mother tongue of science" to better meet pressing current pupil needs.

There would seem to be only the two fundamental needs, which have so often been stated, of the mathematics program of our schools in this state; briefly these are:

1. The need for more effective teaching of arithmetic fundamentals to meet the loudly vociferated needs of the armed forces, industry, business, the colleges, and the more quiet needs of life itself. Each student who is graduated from our high schools should be adequate in fundamental arithmetic skill and in knowledge of the fundamental arithmetic processes.
2. We also would seem to need sounder specific mathematical subject matter for those students whose futures lie in homemaking, industry, nursing, and several other areas. This need implies that we must eliminate some of the material, and strengthen other material, in all mathematical areas.

It should not be necessary to justify either of the two statements above; on the one hand the necessity felt by the armed forces universally in our land (which forced the establishment not only of special in-service courses in simple mathematics for military personnel) but also the insistence by the services upon an increased and new emphasis upon mathematics in general, and arithmetic in particular in our high school pre-induction course should be pointed enough criticism of our failure to do our jobs right in the first place. As to our second point, the certainty so frequently expressed by engineers, scientists, and even the teachers of our subject, that much of the material in our college-type courses is irrelevant and unused deserves at least the expenditure of some energy in definite re-evaluation. Several of the newer texts in geometry are offering a new emphasis in attempting to teach inductive logic and one incomplete phase of what might be called the science of thinking. Several of the newer general texts deriving from preinduction needs, combine arithmetic, algebra, certain needed parts of geometry, and an introduction to the calculus as well as practical applications of these, in one volume. Certain of the publishers, at least, seem to sense a new tendency in mathematics teaching, and seem to be basing their text books on new principles.

The principles inherent in the Connecticut approach to what we feel is an inevitable redirection and adjustment of high school mathematics offer several interesting considerations. Based upon the two needs stated above, the thinking in the Connecticut program seems to begin with something like the following major principles:

1. Mathematics in the secondary school should almost always be a means to an end and not and "end" in itself.
2. Mathematics, as one of the "core" skill subjects, should be offered to every student through every year of the six junior and senior high school years.
3. Some of the more important minimum essentials of "mathematics" outcomes in grades 8–9–10 are:
   a. accuracy and speed in the simple arithmetic operations.
   b. full appreciation by each student of the absolute necessity for mastery of the mathematical fundamentals in any vocational area.
   c. such practical mathematical knowledge in the areas common to all Americans like home and family living and general occupational life as:
      (1) "Mathematical literacy": sound and reasonable concepts of quantity and number, the ability to understand statistical tables and graphs in general reading and
      (2) "Mathematical adequacy" in home bookkeeping and budgeting, common measurement, the mathematics of debt, credit, investments, the laws of probability, etc.
4. Mathematics and arithmetic adapted to specific vocational needs should probably be offered under normal conditions in grades 11 and 12 only.
5. Every subject teacher should be a teacher of the mathematics necessary in his or her field when it is required in the progress of the students—*i.e.*—woodworking mathematics should be normally taught by the woodworking teacher in the woodworking class.
6. The addition of the more useful, modern, and local applications in the objective areas of homemaking, health, vocations, club membership, etc., and the elimination of that material which is unnecessary and outdated would seem plainly sensible.

May we stop for just a moment to reexamine number three above. The implication, of course, is that drill and practice to the point of mastery are required. There is nothing new here, but to my own knowledge there is but one high school in this State which has rebelled against the arithmetic inefficiency of its students sufficiently to provide a practice period in arithmetic each day for all students. We who teach mathematics appear to give lip service to the idea of drill, but we seem to lack the honesty and industry required to do and to have done the job well.

Of course we have been and are impeded in our progress toward a more effective general program in mathematics by certain traditional tyrannies which are stated in the redirection bulletin. Briefly, these are:

1. The tyranny of teaching mathematics as an inflexible body of knowledge and as a "subject" independent of its purposes; of almost always segregating mathematics to a place and time divorced from its primary usefulness.
2. The tyranny of teaching a mathematics dominated by one or two textbooks, which must tend to be inflexible and general rather than local in details.
3. The tyranny of a course of study in mathematics based not upon the realization of objectives, but upon the arbitrary assumptions that 45–55 minutes of each of 180 days, and that 5 class "performances" each week instead of two periods or six, are neces-

sary or desirable. We also seem to assume that any new organization of mathematical knowledge is impossible for such reasons as possible increase in pupil load. For instance, why couldn't a reorganized college-type high school mathematics, integrating as a continuous organized science, our present compartmented algebra, geometry, and trigonometry, offer hope of as much in less time, and in a much more interesting and absorbable way?

A rather interesting prospect offers itself if we contemplate the possibilities once our teaching in mathematics is no longer dominated by the three ideas listed above. We should then have freedom to reach directly for a series of first general and then more practical objectives, using a number of textbooks, a mimeographed outline, or a flexible series of notes and problems; all of this in the place, and the field, where the mathematics is used. We would moreover be able to insist that each pupil have mathematical skill in the area (*i.e.*, homemaking, minimum fundamental essentials, a vocation) where his skill is to be useful and used, and where any real competence demands it.

Last, we should then have an opportunity to teach mathematics for however much time is honestly necessary to an individual or to an area of living: be that three periods a week in the 8th, 9th, and 10th grades and two a week in the 11th and 12th, or what you will.

Please note that nothing in the above is in the nature of a requirement for any school for inclusion necessarily at any time or by any stated date. It is, however, our feeling that some program in mathematics rather like the above, may evolve in some Connecticut high school because the trend, rising from present shortcomings, needs and failures, seems to us in the general directions cited above.

Much to improve our offerings in mathematics can, of course, be accomplished at this time without any interference whatever with our general school programs. Three plans of attack suggest themselves.

First, we ought perhaps to make a beginning of bringing a stronger mathematics to the subject areas, where and when the knowledge and skills are required. One method of procedure might be to bring together the teachers involved in the given field (for example, homemaking) with us who teach mathematics. Our purpose, to see that all of the homemaking mathematics and arithmetic needed is taught and taught well by either a homemaking or a mathematics teacher. Any area in the school which include a major school objective could be appropriately selected, and the choice ought to be made in the school itself by the teachers and staff of the school. This procedure would begin to bring some of our tools to the places where they are to be used—simply and directly. Homemaking, for instance should be the logical place most effectively to teach the mathematics of buying a house, of home bookkeeping, and so on.

A second step very necessary at this time in the opinion of many educators is in the direction of pruning the surplus plumage within our own department. One Connecticut school is at this time planning a condensation of plane geometry and advanced algebra into one year.

One final suggestion may prove of interest. We can in most schools, with direct profit to our students, fairly easily initiate a daily practice period, for *all* students, in the arithmetic and mathematical fundamentals, guided by frequent tests; to include also practice to develop the quantitative judgment and the general mathematical conceptual appreciation of all of our students.

The unpardonable sin is to continue to graduate young people who are mathematically inadequate.

A Mathematics Playlet

By Members of the Mathematics Club
Our Lady of the Lake High School, San Antonio, Tex.

Costumes

Father Mathematics: (*Grandfather*): *An old man with a long, gray beard, wearing an ordinary suit. While on the stage, he dons a large black cloak and a wizard's hat, both decorated with mathematical symbols.*

Sally: *Usual attire of a high school girl.*

Mother: *Low-heeled shoes, hose, house dress, and apron.*

Nancy: *Sandals, short dress, a big ribbon in her hair.*

Artist: *Smock, tam, with palette and brush in his hand.*

Pupil: *In smock also, but with pencil and sketch pad in his hand.*

Hydrogen Twins: *Ordinary school attire, wearing a placard with large plus sign on it.*

Mary Oxygen: *Ordinary school attire, wearing a placard with a large negative sign on it. The back of her placard has "H_2O" in large black letters.*

Mike and John: *Trousers with one leg rolled up, scuffed shoes, shirt tails out, carrying toy guns.*

Pilot, Navigator, Waist Gunner, Bombardier: *No costume needed since they speak over loud speaker from off stage.*

Dancers: *Two dancers for "The Minuet" two for "El Jarabe Tapatio," and one for "The Chinese Sketch"—all in appropriate costumes.*

Miss Rhythm: *Wearing a short, white dress to which are pinned musical notes cut out of black paper, and carrying placards on which are written in large black numbers:*

3 8 4
8 8 4

Tulip Leaves: *Long, white dresses, with a large green crepe paper leaf pinned to the front of the dress; the leaves slant in opposite directions.*

Oak Leaf: *Person rounded out with pillows from shoulder to knees; pillows covered with white sheet; large oak leaf cut from green paper pinned to the front.*

Pill Bug: *In black jacket and long black pants.*

The Stem: *Long, white dress with a long, narrow stem made of green paper, pinned to the front of dress.*

The Tulip: *Wearing long white dress and crown made as a tulip flower.*

The "Pyramid": *Six girls in gym suits.*

Setting

Land of Mathematics. Stage is bare except for a desk and chair on the left and toward the front; another chair is toward the center and back. When Grandfather becomes Father Math, the stage becomes the land of Mathematics by the sudden appearance of solids as stage decorations. This is accomplished by pulling them onto the stage from the opposite wings. At the same time the decorations for the back wall are lowered into place. These are a large framed picture of da Vinci's "The Last Supper" and a large poster which reads: "Big Bargain! Bonds valued at $25.00 now $18.75. Hurry! Be one of the first!"

Sally is sitting at the desk, studying Algebra. Grandfather is reading the paper.

Sally: Oh, how I hate this. I wish I didn't have to study math. These factor problems are terrible!

Grandfather: Now, now, dear! What is the matter? Why math is a wonderful subject. I use it continually. I see it in everything about me—in electricity, in the radio, in my business. Why, math is an essential part of our life. Look, Sally, I've an idea. Let's go into the land of imagination and I will be—now, let's

see. (*Pause*) Oh, I'll call myself "Father Math," and you will be—just plain Sally. (*Dons black coat and wizard's hat, as he speaks. These are handed to him from off stage.*) And then as we travel, I'll show you the value of mathematics in everyday life. Since you will be taking Geometry next year, I want you to appreciate that subject as well as the Algebra you are now studying.

SALLY (*Disgustedly*): Oh, all right, grandpa,—as you say. (*They walk to the right side of stage.*

FATHER MATH: Which use of math would you like to see first?

SALLY: Well, what about seeing it in the home first?

FATHER MATH: All right, Look! (*Back curtain opens, showing mother busy about her baking.*)

NANCY (*entering from the left*): Mamma!

MOTHER: Yes?

NANCY: Can I have a cookie when you finish?

MOTHER: Yes, but run along now so I can finish. (*Nancy exits to left.*) Oh, dear! While she was talking I put in a tablespoon of baking powder instead of a teaspoonful. I guess I shall have to start again. Nancy! Nancy, do you have my $\frac{1}{8}$ measuring spoon? I need it very badly; I cannot go on unless I have it.

NANCY: Yes, mother.

MOTHER: Well, where is it?

NANCY (*enters, looking ashamed*): I was making mudpies and lost it. I don't see what you need with it anyhow. It's too little to do any good.

MOTHER: Now what shall I do? I can't go on baking then, because without it I can't get that accurate a measurement. I think I shall have to give up my baking attempts for today. (*Looks at watch*) It is so near lunch I won't have time to try again. I'll try some other time when I won't have to be a mathematical genius to bake cookies! (*Back curtain*)

FATHER MATH: Like it?

SALLY: Yes, grandpa—er, Father Math.

FATHER MATH: Now let us take a look at Mathematics as applied to art.

(*Enter pupil and artist from the right wing and walk toward the left side, as the picture of "The Last Supper" is on the left side of the back curtain.*)

PUPIL: I don't understand why you have to bring up Math in art. It is so boring, and how could it have any connection with art?

ARTIST: Here, I will explain. Take this picture (*pointing*) of "The Last Supper" by da Vinci. The depth of the room is clearly conveyed, because da Vinci has drawn the walls on the left and right converging and not parallel; he did the ceiling and floor the same way. You should note, too, that if the lines of walls, floor, and ceiling were to be extended, they would all come together at the head of Our Lord, Who is the center of interest in this picture.

PUPIL: But what does that have to do with math?

ARTIST: Lots! You learn in Mathematics how to make something look deep when it isn't. If da Vinci had drawn the lines of the walls parallel, it would not look like a room; there would be no perspective, and Our Lord would not be the center of everything in the picture.

PUPIL: Oh, I see. In other words, geometry and art are related subjects. So even though I dislike geometry, it is still a wonderful thing. (*Exit artist and pupil to the left.*)

SALLY: I hope I'll like geometry next year.

FATHER MATH: Sally, I've been wondering. Do you take chemistry?

SALLY: Oh, yes, but please don't remind me of that!

FATHER MATH: Well, have you had any math in it yet?

SALLY: Yes, sorta.

FATHER MATH: I'll tell you what. Let's let someone in the land of imagination show you one important theory in chemistry —that like particles repel each other and unlike particles attract. (*Pause.*) Here come the Hydrogen Twins (*fighting*). I wonder what they're up to. Shhh! I see

Mary Oxygen too (*Enters*). Why, look at the twins run over to her. They stick together like—Hmmm! Why, look at that! H_2O! (*As Mary Oxygen enters, she takes her place between the Hydrogen Twins, and then smilingly reveals the back of her placard which says "H_2O." They then exit arm in arm.*) Did you see that, Sally? Did those "+'s" and "−'s" remind you of anything?

SALLY: Oh, yes! That was interesting. I am beginning to like math. What are we going to see next?

FATHER MATH: Oh, the business world! Surely, Sally, you can tell me about that.

SALLY: You mean bankers—and merchants—and—and—

FATHER MATH: Well, shall we say also statisticians, doctors, postmasters, government officials in internal revenue offices—just to mention a few others. Let's see one result of the banker's work—something of great value to us now.

(*Two little boys, Mike and John, stroll onto the stage from the left. One has a market basket; the other has a purse with money. The poster on the back curtain catches their attention.*)

MIKE: Say, John! Look at that. A bargain! Mother will like that.

JOHN: Ah, Mike, that gives me an idea. (*Then he whispers into Mike's ear. They open the purse and start counting the money.*)

MIKE: That's wonderful! Mother is going to like that. (*They run off the stage. After a short pause, they come back holding a bond.*)

MIKE: John, do-o—you—think—mother is going to mind?

JOHN: Heck, no. She's going to be proud of us.

MIKE: I hope you're right.

(*Enter mother from the right, sweeping.*)

TOGETHER Mother! Mother! We have a surprise for you.

MOTHER: Did you get my groceries? Did someone get hurt? Have you got the change? Well, tell us what happened?

TOGETHER: We—we bought you something.

MIKE: You know that big sign down the street? The one says: "Bargain." Well, look, mother. We got you a bond (*holding it up*).

MOTHER: You got me a bond? How? With what money?

JOHN: Well, mother, you see—we thought you wouldn't mind.

MOTHER: (*After a short pause.*) Well, boys, since you did it for our country and since it really is a bargain, I won't spank you, but mind you—(*They all three walk off, mother still talking.*)

SALLY: Gee, I didn't know even bonds were put on sales.

FATHER MATH: Well, not exactly! Let's say rather that it's compound interest that makes an $18.75 bond worth $25.00 in ten years. Can you figure that out?

SALLY: Who? me? You know better!

FATHER MATH: And now, Sally, I've a surprise for you. Oh, say, I hear a plane coming. Since we are in the land of imagination, let's suppose we were up there and you are the co-pilot of a B-17 (*Flying Fortress.*) I think the plane is headed for Germany—right for Berlin. You listen to the conversation between the crew members of the "Home, James." (*The members of the crew speak from off stage over a loud speaker.*)

PILOT: Pilot to navigator. Over.

NAVIGATOR: Navigator to pilot. Over.

PILOT: No trouble in sight, Mack, but request position, altitude, temperature, air-speed, ground speed, wind direction and velocity, and distance to target.

NAVIGATOR: Roger, *Wilco* (*Means will comply with the request.*)

PILOT: Roger.

NAVIGATOR: Navigator to pilot. Over.

PILOT: Pilot to navigator. Over.

NAVIGATOR: At our present altitude of 25,000′ the temperature is −20° Centigrade. We have a direct tail wind from 210° at 46 miles per hour. Our true

heading is 30° and with a zero degree drift correction our true course is also 30°. With our present ground speed of 271 miles per hour, we should be over Scheildkritz, Germany, in exactly four and one-half minutes.

PILOT: Roger. (*About ten seconds later*) Pilot to Right Waist Gunner! Focke-Wulf-190 coming in at three o'clock high. Watch him!

R. WAIST-GUNNER: Climb a little. I have him now. Come on in Nasty Nazis and join the party! (*Pause*) I got him! I got him! Chalk him up for the books. Don't see any others, do you?

PILOT: No! Must have been alone! They try that sometimes.

BOMBARDIER: Congratulations! Good shot, Eddie!

NAVIGATOR: Fine work, Ed. The drinks are on us—you really saved our lives.

PILOT: We are over Scheildkritz now. What's the score, Mack?

NAVIGATOR: From here to Berlin on our present heading is 63 miles. We will be there in fourteen minutes. This is the Initial Point (I.P.) of our bomb run. (*Pause*)

BOMBARDIER: Target sighted and identified. Bomb bay doors coming open. After bombs away we will turn to the right very sharply so that we can avoid the heavy "flak" section of Berlin.

BOMBARDIER: Bombardier to pilot—correct 5° right and hold course for 30 seconds. *Steady*! *Steady*! *Bombs away*!

SALLY (*after slight pause*): Oh, that was exciting! I want to know math so that I can perhaps some day be a pilot; it's always been one of my ambitions.

FATHER MATH: Sally, I'm sure you realize the part mathematics plays in the war, but a good many of the boys and girls going into service don't have enough math education, examiners tell us. Take, for example, the business arithmetic knowledge the men and women of the Quartermaster Corps and Troop Service Command must have so as to be able to deal with the supplies of food, clothing, housing equipment and ammunition, for the Post Exchange and Commissaries also carry on a great many business dealings. Another phase of the business mathematics field is the selling of bonds and stamps. The different people who have to handle the money and bonds must be competent from the kid who sells stamps to the paymaster giving out the pay to the Private 1st Class. Since the first days of the compass, plane geometry has been the biggest factor in guiding ships safely through the seas. Now also geometry and trigonometry guide men through the skies. But that's enough explanation. Now what would you like to see?

SALLY: Well, is there any math in music?

FATHER MATH: Now, suppose you tell me since you've taken music for two or three years.

SALLY: What do you mean? I have studied theory which tells us that there are certain number of beats to a measure. Sometimes there are dotted notes which add ½ the value to the note.

FATHER MATH: Is that all?

SALLY: Well, there are different time values; as, $\frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \frac{6}{8}$, just to mention a few.

FATHER MATH: Yes, that's what I mean. Let's see that applied to dancing.

(*Miss Rhythm steps on from the left. She displays the placard which reads* $\frac{3}{4}$ *"4" during the "Minuet" and the other two placards during the respective dances. Each dance lasts only a minute or two and follows the preceding one in rapid succession.*)

SALLY (*dancing*): Oh, say, that was fun! Thank you.

FATHER MATH: Oh, you're welcome, Miss Sally (*slightly bowing*).

SALLY: I want to see more. Let's go! Let's go!

FATHER MATH: All right. Then let's take a peep at the outside world. (*In a puzzled voice*) What has that to offer in connection with mathematics? Oh, I hear a sound. Sh-h! We'll have to be very

quiet to hear the conversation. (*As the back curtain opens, the tulip leaves are seen talking to each other.*)

ONE LEAF: Have you noticed how big we are? Three inches long.

OAK LEAF (*stepping on*): Hmmmm! Why, that's nothing. I am five inches long and five wide—or better known as Mr. Five by Five.

PILL BUG (*crawling on*): But I can show you all up. (*Takes a big bite of the Oak Leaf and then rolls up.*) Look at the perfect circle I am!

THE STEM (*Strolls on*): Hey, what's going on here?

T. LEAVES: Stem, we have something to ask you.

THE STEM: Well, what is it?

O. LEAF: Do you have any math in you?

THE STEM: Why, sure. I am ½ inch in circumference and 14 inches long. See, I am a big stem. (*Pause*) Well, lookie here! Here comes the Tulip. She seems to have something to tell us about herself. (*Enter tulip.*)

THE TULIP: Indeed I do! Haven't you ever noticed all my lovely petals? They form a hexagon, since there are six of them. Just take a look at me. (*Struts. Meanwhile the tulip leaves take their places on each side of the stem. Oak leaf exits.*)

OTHERS: Beautiful! Come and join us. Won't you? (*She takes her place behind the stem, so that the picture of the flower is complete. Pause and then close back curtain.*)

SALLY: Oh, how I do love flowers, and now I can appreciate them even more.

FATHER MATH: Do you want to see more or would you rather go home?

SALLY: Well, what else is there?

FATHER MATH: Now's let's see. You've seen types of buildings and bridges. Even your home is a type of architecture. Let's look at some other forms of architecture. (*Back curtain opens enough to reveal an arch.*)

SALLY: Oh, look at that arch! Isn't it beautiful?

FATHER MATH: A masterful piece of work, indeed. But did you ever stop to think that math is used in even so simple a structure? Geometry has played a very important part in the construction of that lovely arch. Notice how balanced it is,—how one side is just like the other. Mathematicians call that symmetry. Now let's see just one more type of architecture—a kind we are all familiar with—the pyramid. (*Six girls in gym suits enter by way of the arch and form a pyramid. Sally shows her approval by clapping. Girls exit.*)

FATHER MATH: But now, Sally, we must really go! (*Starts to walk toward the center of stage.*) By the way, Sally, do you still hate mathematics?

SALLY: No, grandpa—er, Father Math. I'm ready to go back and finish that homework.

(*Enter all characters from right and left wings, forming a double semi-circle about Father Math and Sally, and sing:*)

Tune: Auld Lang Syne

Should all math'matics be forgot
And treasures that it hath?
Should all math'matics be forgot
And days with Father Math?
For Fath-er Math, my dear,
For Fath-er Math,
We'll think and learn and reason, yes,
For Fath-er Math.

CURTAIN

Notice!

On page 274 of the October issue of this journal the equation

$$ds = r\sqrt{1+\sin^2 u\left(\frac{dv}{du}\right)^2}\,du$$

should have been

$$ds = r\sqrt{1+\sin^2 u\left(\frac{dv}{du}\right)}\,du$$

Aerial Navigation as Applied Mathematics

By LAURA BLANK
Hughes High School, Cincinnati, Ohio

IN THE FALL of 1942 at the urgent request of the federal government, as an incentive to interest in piloting and navigating airplanes, many of the secondary schools of our country set into operation classes in aerial navigation, aerodynamics and meteorology. The navigation courses were in the main, the responsibility of the teachers of mathematics. They have been preparing youth including a few young women now for three years. The motive of these young folk in selecting the course is either the wish to pilot soon or that of understanding a timely subject. Many of the young men of the earlier classes are flying missions now "down under" in the South Pacific.* Some are flying their own planes from England for furloughs and then new assignments. Now the Army Air Corps is closed to admissions, and moreover, many men classified in that branch of the service for months, "on the line," awaiting anxiously their transfer to preflight have been notified officially that they will not be needed as pilots or navigators or even bombardiers. One wonders what the effect will be on elections to a high school course in navigation. Will it develop that aerial navigation is an emergency subject, incident to the war, in secondary schools, to vanish from the curriculum in a few years, parts of it to be taken over into the courses in so-called pure mathematics? Or will navigation continue as a course optional in high school? At the moment options are holding up well.

Such considerations, on the part of instructors whose basic interest is mathematics, lead to an appraisal of the subject, classified as it usually is taught, as applied mathematics. What kinds of mathematics are needed in an elementary, but thorough course in navigation? Is the necessary mathematics, used in air tight compartments? Is it quite elementary? Or quite advanced for the secondary school level? Does this subject, though a somewhat restricted field, yet bringing with it a tremendous spontaneous interest, "fuse mathematics" as perhaps few, if any high school subjects do?

First of all, to consider preparation, a pupil should have a thorough grounding in arithmetic, algebra and plane geometry. The solid geometry necessary can be taught informally in the navigation class, just as it is needed. Thus it will be assimilated gradually.

The arithmetic needed in the course is of all sorts, the four fundamental operations, the use of common fractions, decimals and percentage, and constant use of sexagesimals in time and circular measure. The need of arithmetic is at all times and in all sorts of connections.

In algebra, the basic formula is $r \cdot t = d$, rate multiplied by time equals distance, assuming uniform speed. It is used in its three forms, solving now for distance; again, time; at another time, for rate. Proportions are much used. For example, how long will it take to fly 120 miles, if a plane flies 3 miles in 52 seconds? Again, if an inch on an aeronautical chart, represents 16 miles, $7\frac{1}{2}$ inches depict how many miles? Again: How would distances between corresponding points on two charts drawn on the same projection compare, if one chart is constructed on the ratio 1:500,000 whereas the other is on the ratio 1:1,000,000?

Another much used formula is the radius of action formula:

$$RA = t\left(\frac{GS_0 \cdot GS_i}{GS_0 + GS_i}\right)$$

By radius of action is meant the greatest distance an aircraft can fly out from a given base, then without landing, return

* This paper was written in the early summer of 1945.

to the same base, yet remain within a given limit of fuel or daylight hours, having previously made allowance for an emergency reserve of either fuel or daylight, as the case may be. Regularly all of the variables in the second member of the equation can be found. The formula can be made convincing to a class, as it can readily be derived by several methods, by the instructor. A variant of this formula is that of radius of action to *alternate* base:

$$RA = \frac{t.R_a}{R_d + R_a}.$$

In the problem of radius of action to alternate base, the question is to find how far an aircraft may approach a given destination along a predetermined course, and in case of the pilot's learning, by radio en-route, of bad weather conditions, at the destination, yet turn off to an alternate base, arriving within his given fuel supply or hours of daylight. Again, the variables of the second member of the formula are readily obtained.

In navigation it is geometry, plane and solid, which we have need of always. To be sure, it is not all of what we recall, whenever we say "plane geometry" or "solid geometry," but perhaps some of the most useful parts in general, of both subjects. In the former, it is angles in all possible positions and connections, and parallelograms, with all the theorems concerning them. Also there is the theory of vectors, vector triangles and parallelograms of forces, basic to all navigation, where the motivation is a combination of that produced by power plant and wind. The drawings are meticulously constructed to scale, and with a protractor, and the conclusions interpreted by the same instruments. This method carries over to a navigation computer, Dalton Mark VII or E-6B, which are most used in practice, but which would be completely incomprehensible without the basic visualization acquired through the carefully scaled geometric drawings. These computers carry, too, circular slide rules for shortening arithmetic computations, multiplication and division and combinations of them. It is not necessary for the pupil to know the theory of exponentials and logarithms upon which the slide rule is constructed in order to use it. However if time permits, it is better for him to understand it so as to make his use of it clear and convincing.

Back of the whole theory of navigation is spherical geometry. The pupil need not glibly quote theorems or even be able to prove them, but he must be able to visualize on a sphere, its great circles and small circles, its axis, poles, equator, prime meridian, other meridians, parallels, the spherical co-ordinates of a point, and also the rotation of the earth, and consequent time. He must be able easily to visualize a right circular cylinder tangent to the sphere along its equator, or another great circle, then to project the network of meridians and parallels, from the center of the sphere, as a projection point, onto this cylinder. This cylinder cut along an element and flattened, the Mercator projection, is one kind of plan for chart construction. Next, he must be able to call up the mental picture of this mesh of meridians and parallels projected onto a right circular cone tangent to the sphere along a small circle, thus producing the plan of the conic projection. Next, he should visualize the cone as intersecting the sphere along one parallel, say the 45th° N, and coming out again along another, possibly the parallel 33rd° N, then using a portion of the cone, truncated near these parallels, he has an idea of the cartographer's "Lambert conformal conic" projection. There are also charts called "polyconic" constructed on the plan of many strips from cones, each tangent along a different parallel. Finally the gnomonic or tangent plane projection is the newest and the one of the greatest interest for shortest distances, as all great circles project in straight lines on such a chart, the great circle being the shortest distance from one point to another on our earth. A navigation pupil must be able to visualize the schematic reproduction of

parallels and meridians on each such projection. He must learn to understand the shortcomings and merits of each such plan, for each has both sorts of quality. He should know them so well that he becomes competent to judge the suitability of one or another for the navigation problem at hand. He should realize that a spherical angle is the same in size as the simpler plane angle formed by one line tangent to each arc of a great circle forming the spherical angle at their vertex.

There are many scientifically constructed tables and graphs of information useful to the navigator or pilot-navigator. Celestial navigation, highly mathematical, is rarely regarded as elementary, at the high school level. Radar navigation in civil flying, will be a unit of study for postwar classes. Little scientific information concerning its use is released, at the present time.

This by no means summarizes all of the examples of mathematics used in elementary navigation, but one wonders if one could work out a more interesting and better unified refresher course in mathematics. There is nothing artificial or forced in the dipping into and use of subject matter of the various fields of elementary mathematics. Truly it is not a complete review of all that a pupil might somewhere at some later time need of his earlier experiences in mathematics, but is anyone wise enough to outline such a course? The geometry used in this course was never so well understood and applied in the course in pure mathematics studied prior to navigation as at the end of the latter course. All in all, the study of navigation in high school has been one of our most interesting, unanticipated, recent psychological and pedagogical experiments "plucked out of the blue" in the field of mathematics.

The Influence of Objective Measurement

One administrator makes this statement about the use of standard tests: "Our greatest advance has been in the development of a new point of view—in the realization that since differences exist we must do something about them. . . . The influence of objective measurement of ability and achievement is apparent everywhere. It has given us the basis for organization of new kinds of classes, for refinement in classification, for continuous readjustment of the school program—the beginning of our efforts to adapt the school to the variety of human abilities."

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The Irrationality of $\sqrt{2}$

By HOWARD EVES
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1. Introduction

ONE OF THE milestones in elementary mathematics is the establishment of the existence of irrational numbers. This is usually accomplished by showing that there exists no rational number x such that $x^2=2$, thus proving that $\sqrt{2}$ is irrational. This particular irrational, the discovery of which is ascribed to the early Pythagoreans, was perhaps the first irrational number to be known, and probably arose in considering the ratio of a diagonal to a side of a square.

The following is elementary and was prepared for presentation to an undergraduate mathematics club. It gives a few alternative ways of proving that $\sqrt{2}$ is irrational, illustrating diversity of mathematical attack and, at the same time, some historical methods.

2. Proof by Elementary Number Theory

We first observe that, for a positive integer s, s^2 is even if and only if s is even. Now suppose $\sqrt{2}=a/b$, where a and b are relatively prime integers. Then

$$a^2=2b^2. \tag{1}$$

By (1) we see that a^2, and hence a, must be even. Put $a=2c$. Then (1) becomes

$$2c^2=b^2,$$

from which we conclude that b^2, and hence b, must be even. But this is impossible since a and b are relatively prime.

3. Another Numbers Proof

A. *Fundamental theorem of arithmetic.* Given any positive integer a, there are unique non-negative integers $\alpha_1, \alpha_2, \alpha_3, \cdots$, only a finite number of which are different from zero, such that

$$a=2^{\alpha_1}3^{\alpha_2}5^{\alpha_3}\cdots,$$

where 2, 3, 5, $\cdots$ are the consecutive primes.

B. *Notation.* Let $a=2^{\alpha_1}3^{\alpha_2}5^{\alpha_3}\cdots$. Then we write $a=(\alpha_1, \alpha_2, \alpha_3, \cdots)$.

C. $\sqrt{2}$ *is irrational.* Suppose $\sqrt{2}=a/b$, where a and b are positive integers. Let $a=(\alpha_1, \alpha_2, \alpha_3, \cdots)$ and $b=(\beta_1, \beta_2, \beta_3, \cdots)$. Then we have, since $a^2=2b^2$,

$$(2\alpha_1, 2\alpha_2, \cdots)=(1+2\beta_1, 2\beta_2, \cdots),$$

whence

$$2\alpha_1=1+2\beta_1.$$

But this is impossible.

4. Proof by Fermat's Method of Infinite Descent

A. *Method of infinite descent.* In outline the method is this. To prove that there do not exist positive integers $p, a, b, \cdots$ satisfying a relation $R(p, a, b, \cdots)$ assume the contrary. On this assumption show that $R(p_1, a_1, b_1, \cdots)$ holds, where $p_1<p$. Then in like manner we may show that $R(p_2, a_2, b_2, \cdots)$ holds, where $p_2<p_1$, and so on ad infinitum. But, since there is only a finite number of positive integers less than p, this is impossible. We are thus led to a contradiction, whence we conclude that the relation $R(p, a, b, \cdots)$ is not satisfied by positive integers $p, a, b, \cdots$.

B. $\sqrt{2}$ *is irrational.* Suppose $\sqrt{2}=a/b$, where a and b are positive integers. Now $\sqrt{2}+1=1/(\sqrt{2}-1)$, whence

$$\frac{a}{b}+1=\frac{1}{\dfrac{a}{b}-1}=\frac{b}{a-b},$$

and

$$\sqrt{2}=\frac{a}{b}=\frac{b}{a-b}-1=\frac{2b-a}{a-b}=\frac{a_1}{b_1}, \text{ say.}$$

But, since $1<\sqrt{2}<2$, we have $b<a<2b$,

whence $0<2b-a=a_1<a$. Hence the theorem by infinite descent.

5. Theory of Equations Proof

A. *Theorem* (Gauss). *A polynomial*

$$x^n+p_1x^{n-1}+p_2x^{n-2}+\cdots+p^n$$

with integral coefficients, cannot have a rational non-integral root.

For suppose that the polynomial has a root a/b, where a and b are relatively prime integers, $b>0$. Writing a/b for x, and multiplying by b^{n-1}, we obtain

$$-a^n/b=p_1a^{n-1}+p_2a^{n-2}b+\cdots+p_nb^{n-1}.$$

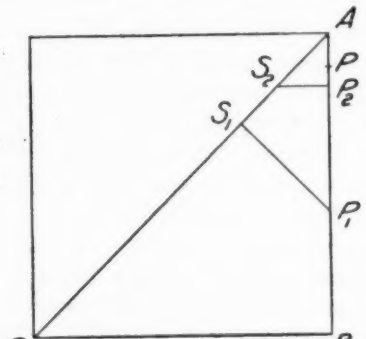
That is, a^n/b is an integer. But a and b are relatively prime, whence we must have $b=1$.

B. $\sqrt{2}$ *is irrational.* Set $x=\sqrt{2}$. Then $x^2-2=0$. Clearly no integer satisfies this equation. Hence, by Gauss's Theorem, the equation has no rational root.

6. Geometrical Proof

We shall give a geometrical demonstration by showing that a side and diagonal of a square are incommensurable.

Suppose the contrary. Then (see figure) there exists a segment AP such that both AB and AC are integral multiples of AP. We shall say, then, that AB and AC are commensurable with respect to AP. On CA lay off $CS_1=AB$ and draw S_1P_1 perpendicular to CA. Then AS_1 and



$AP_1=AB-AS_1$ are commensurable with respect to AP. On P_1A lay off $P_1P_2=S_1A$ and draw P_2S_2 perpendicular to AB. Then AP_2 and AS_2 are commensurable with respect to AP. But we may continue this process until we have AP_n and AS_n commensurable with respect to AP and $AP_n<AP$. But this is absurd. Hence the theorem.

Mathematics and Its Relation to the Life of Man

By Mabel Stewart, *Central High School, Oklahoma City, Okla.*

I. *Man* has a mind.—*Mathematics* is a great adventure in logical and precise thinking.

II. *Man* must find a place in the economic world.—*Mathematics* is the basis for many trades and professions.

III. *Man* has a place in history.—A part of his inheritance of the past is his knowledge of the part *mathematics* has played in the progress of the universe.

IV. *Man* has cultural needs and must fit himself into the cultural pattern of the world.—All art, music, painting, poetry, sculpture and architecture, has its foundation in *mathematics*.

V. *Man* is a moral creature. *Mathematics* is a constant object lesson in mental integrity and intellectual honesty.

Verbal Problems—Again!

By JOSEPHINE WIBLE
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THIS ARTICLE is written to suggest to other teachers of high school algebra a possible way of compromising with those ever-present students in an algebra class who maintain that it is ridiculous to be obliged to solve a verbal problem by algebra when it can be solved more quickly and easily by arithmetic. To be sure, no one can say when or where arithmetic leaves off and algebra begins; but we well know what a pupil means when he insists on the arithmetic procedure.

As teachers, we cannot deny that a considerable number of so-called verbal or story problems assigned in algebra classes can be solved more economically without any apparent use of algebra. Many a teacher solves such problems mentally, using the very method she is all too likely to refuse to accept from her pupils because it seemingly is not algebra. This smacks of unfair play; but too often the teacher sees no way to reconcile the algebraic and the arithmetic solutions and feels duty-bound to insist on a long algebraic solution.

For example, suppose the problem is:
A machine can do a certain job in 5 hours. A newer type of machine can do the job in 3 hours. How many hours are needed to complete this same job, using both machines?

When most teachers solve this problem, for the sole purpose of obtaining an answer, they cannot resist the temptation of using what might be called pure arithmetic. They think that the number of hours $= \frac{5(3)}{5+3} = \frac{15}{8} = 1\frac{7}{8}$ hr. In so doing, why should we feel guilty? Why should we (eventually, at least) refuse to accept this procedure if proposed by a pupil? In fact, are we not amiss in our teaching of algebra unless some pupil does eventually propose it? Yet, too often we frown and object when a pupil presents such a brief bit of work. We insist that he must use letters, some way or another, since he is in an algebra class.

The author is postulating two statements:

1. It is our duty, as educators, to encourage the habit of generalization
2. Algebra is a powerful and dependable instrument for generalizing when dealing with mathematical situations.

I am well aware of the emphasis we should place in mathematics classes on the danger of making hasty generalizations. I am reminded of Gwethalyn Graham's accusation in "Earth and High Heaven" that nowhere does our education warn us sufficiently of the dangers of hurried generalizations. But surely we should encourage the use of algebra to arrive at generalizations which are not hasty and which are valid. No one subscribes to the tenet that algebra is merely a tool but to refuse to use it as a tool, for generalizing purposes, would be deplorable.

Returning to the so-called "work" problem, previously stated, let us generalize, via algebra, letting t_1 and t_2 be the given number of hours instead of 5 and 3, respectively.

Let x = no. of hrs. required to complete the job when both machines are operating

$\frac{1}{t_1}$ = part of job done by old machine in one hr.

$\frac{x}{t_1}$ = part of job done by old machine in x hr.

$\frac{1}{t_2}$ = part of job done by new machine in one hr.

$\frac{x}{t_2}$ = part of job done by new machine in x hr.

$$\frac{x}{t_1}+\frac{x}{t_2}=1$$

$$x(t_2+t_1)=t_1t_2$$

$$x=\frac{t_1t_2}{t_1+t_2}$$

(The same result is obtained by solving $\frac{1}{t_1}+\frac{1}{t_2}=\frac{1}{x}$.)

It would be unwise to hand out the formula $\frac{t_1t_2}{t_1+t_2}$ to a class when such problems are assigned. My thesis is that, sooner or later, the class should derive this formula and then be permitted to use it. Some pupils are able to reason through the problem without any so-called algebra at all, doing it, as they say, by arithmetic. Surely, we should not penalize them because they are good reasoners or proficient in arithmetic. Yet, we want those pupils to have an appreciation of algebra. The compromise proposed is:

1. the teacher challenges such pupils to substitute appropriate letters for the numbers in a specific problem and solve algebraically
2. the pupil discovers whether or not the general algebraic solution is consistent with the arithmetic procedure used in a specific case
3. if such consistency is evident, the teacher thereafter accepts the shorter solution. In the problem cited, teacher accepts $\frac{5(3)}{5+3}=\frac{15}{8}=1\frac{7}{8}$ hrs., whether or not pupil states the formula $\frac{t_1t_2}{t_1+t_2}$. By arithmetic, the pupil has probably argued that since together the machines do $\frac{1}{5}+\frac{1}{3}$ or $\frac{8}{15}$ of the job in one hr. and $\frac{15}{15}$ of the job must be done, the no. of hrs. required is $\frac{15}{8}$, which is consistent with $\frac{5(3)}{5+3}$ or $\frac{t_1t_2}{t_1+t_2}$.

I doubt if we teachers need to worry about algebra losing prestige merely because we acknowledge that the result can be obtained so simply. On the contrary, the majority of students will see more clearly the power of algebra, will see that it is arithmetic "generalized" and will respect it the more. The algebra will furnish proof that the arithmetic they may have used is correct to use in all similar situations.

I repeat that it seems to me we teachers are at fault if, sooner or later, our pupils fail to derive a formula or general solution. We have failed not only to impress them, sufficiently, with the power of algebra to generalize but we are denying them an easy solution of the problem. It is a rare pupil, although I can vouch that he does exist, who will attempt such generalization on his own initiative. However, as pupils see the value of the resultant formula, they will be increasingly apt to wish to derive formulas.

It is not at all essential that the teacher insist that a derived formula be learned and used. However, it seems reasonable to accept a solution obtained by substituting in the formula. If the problem is a more or less common or practical one which we feel the majority of pupils should know how to solve, it would be economical to learn and use the formula. If the problem is assigned to show the use of algebra or to furnish practice in reasoning or in algebraic manipulation, it has served its purpose after a few other similar problems have been solved and the generalization has been obtained.

Incidentally, it would not be advisable to assign such a problem as the one discussed above to most classes unless such questions as the following had been discussed previously:

(1) If a boy can mow a lawn in 4 hours, what part of the lawn can he mow in 1 hour? In 3 hours? In t hours?

(2) If a press can print an edition of a newspaper in h hours, what part of the edition can the same press print

in 1 hour? In 2 hours? In x hours?

The author realizes that $\frac{t_1t_2}{t_1+t_2}$ is not the solution of all work problems. There are numerous variations of such problems which cause variations in the general solutions.

Other verbal problems are stated below and their general solution given.

An aviator's ground speed with the wind is "r_1" mph but returning against the wind it is "r_2" mph. How far can he fly and yet return in "t" hours? (Radius of action problem)

Let x = no. of hrs. flying out

Then $t-x$ = no. of hrs. flying back

$$r_1x = r_2(t-x)$$

$$x = \frac{r_2t}{r_1+r_2}$$ and hence distance one way is $\frac{r_1r_2t}{r_1+r_2}$ miles

A merchant wishes to mix seed worth "c_1" cents per pound with seed worth "c_2" cents per pound to produce a mixture of t pounds worth "c_3" cents per pound. How many pounds of each kind of seed should be used?

Let x = no. of lbs. @ c_1 cents

Then $t-x$ = no. of lbs. @ c_2 cents

$$c_1x+c_2(t-x) = c_3t$$

$$x(c_1-c_2) = t(c_3-c_2)$$

$$x = t\left(\frac{c_3-c_2}{c_1-c_2}\right)$$ and

$$t-x = t\left(\frac{c_1-c_3}{c_1-c_2}\right)$$

"It is better to change the form of the coefficients of t to $\frac{c_2-c_3}{c_2-c_1}$ and $\frac{c_3-c_1}{c_2-c_1}$, respectively.

The speeds of a boat and an airplane "d" miles apart are "a" mph and "b" mph respectively. How soon will they meet if they start at the same time and travel the same route?

Let x = no. of hrs. before they meet

$$xa+xb = d$$

$$x = \frac{d}{a+b}$$

A boy on a bicycle travels "a" mph. He must be overtaken in "c" hours by a second boy starting "h" hours later. Find rate of second boy.

Let x = no. of mph of second boy

$$x(c) = a(c+h)$$

$$x = \frac{a(c+h)}{c}$$ miles per hour

(Since in this type of problem, $x(c) = a(c+h)$ is analogous to $r_1t_1 = r_2t_2$ or $\frac{r_1}{r_2} = \frac{t_2}{t_1}$, one may teach that the rate varies inversely as the time.)

How many ounces of (1) water must be added to "n" ounces of an iodine solution to decrease the per cent of iodine from "a"% to "b"%? (2) pure iodine must be added to "n" ounces of an iodine solution to increase the per cent of iodine from "a"% to "b"%?

1. Let x = no. of oz. of water to be added

$$.01b(n+x) = .01an$$

$$b(n+x) = an$$

$$x = \frac{n(a-b)}{b}$$

2. Let x = no. of oz. of pure iodine to be added

$$.01b(n+x) = .01an+x$$

$$b(n+x) = an+100x$$

$$bn+bx = an+100x$$

$$bx-100x = an-bn$$

$$x = \frac{n(a-b)}{b-100}$$ or $\frac{n(b-a)}{100-b}$

A different kind of example of the power of algebra to predict or determine what will happen under certain conditions is illustrated by the following problem which arose in our school system not long ago.

Salaries of most teachers in our school system are paid biweekly for nine months. Since the school board must deduct for income tax, what effect, if any, would be produced by a change to biweekly payments for 12 months?

Let s = no. of dollars in the individual teacher's annual pay

$\frac{s}{18}$ = no. of dollars in biweekly pay (9 mo. basis)

$\frac{s}{24}$ = no. of dollars in biweekly pay (12 mo. basis)

Let e = no. of dollars in the individual teacher's biweekly exemption

(At the time this problem arose, one's biweekly exemption was the yearly exemption divided by 24, no matter whether one received 18 or 24 salary payments. This may or may not be true at the present time.)

Since the school board deducted one-fifth after exemption had been deducted,

$\frac{18}{5}\left(\frac{s}{18}-e\right)$ = no. of dollars withheld during entire year (9 mo. basis)

$\frac{24}{5}\left(\frac{s}{24}-e\right)$ = no. of dollars withheld during entire year (12 mo. basis)

$$\text{Difference} = \frac{18}{5}\left(\frac{s}{18}-e\right) - \frac{24}{5}\left(\frac{s}{24}-e\right)$$

$$= \frac{s}{5} - \frac{18e}{5} - \frac{s}{5} + \frac{24e}{5}$$

$$= \frac{6}{5}e$$

Hence, we reach the conclusion that if paid on a 12-month basis we would pay less income tax and that the difference for all individual cases would be $\frac{6}{5}$ of the biweekly personal exemption. (I cannot say whether or not the federal government makes a refund to adjust the difference.) It is evident that the factor determining the difference is not the salary but the exemption.

There is certainly more information and satisfaction gained by using algebra than could be realized by investigating dozens of individual cases, arithmetically; in fact, one might miss the final conclusion altogether if merely investigating individual cases, numerous as they might be.

The time spent in teaching pupils the purely manipulative phases of algebra needs justification. However, all wide-awake mathematics teachers know many, many outcomes that do justify time so spent. The author is, for the most part, emphasizing one, only; namely, the use of letters and algebraic operations to reach a general conclusion.

In summary, urge pupils to generalize in order to derive formulas and short cuts; then accept the short solutions after they have been demonstrated algebraically and thus compromise with those pupils who rebel against a lengthy algebraic procedure for every problem of a certain type.

Algebra

Into algebra class we march on time,
And bubble with gossip till the sound of the chime
(Which comes in the form of the teacher's voice,
So soft, yet stern, we stop without choice).
The class begun, terms then fly,
And everyone looks with an inquiring eye
At the equation on the board in which they see
That x equals y and z equals "could be?"!
Then together they shout in one piercing cry
"We demand to know just why
z equals could be and x equals y!"
Sufficient explanation having been given,
The pupils attack their logarithms,
In which the old method is treated with derision
While the new is more complicated than long division.
Then quadratics come into view,
Which we look upon as nothing new.
But oh, what a struggle is a literal equation,
Which some madman created for his recreation
Yet for algebra, with all its complications,
And indefinable situations,
Not one-thousandth of a cent
Would I give for all the hours I've spent
Over a problem that seemed but one continuous labyrinth.

Written by Ruth Panzer
William Penn High School
Submitted by E. Levin
Department of Mathematics

A Training Course for Engineering Aides*

By JULIA WELLS BOWER
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THE DEARTH of engineers occasioned by the war has encouraged the use of aides for handling engineering routine. The skill which these young women have demonstrated in assisting with experiments, making slide rule calculations, plotting charts and performing other similar duties has given rise to speculation concerning the likelihood of engineering aide work becoming a recognized occupation. If this new vocation is established, it will probably remain in the hands of women. This discussion of the course given at Connecticut College to train aides for the research division of a Connecticut aircraft corporation is therefore a timely one.

The training was first given during six weeks of the 1943 summer session. Enrollment consisted of approximately a dozen college graduates who had studied at least one year of college mathematics. The students were chosen by the corporation and entered its employ at the beginning of the course. They were instructed in the fundamentals of aerodynamics and thermodynamics so that they would understand the significance of their assignments. They became proficient in the use of the slide rule so that they could make desired calculations. They were introduced to engineering drawing so that they would be properly literate in this important language of symbols. They were drilled in uniformity in lettering and accuracy in curve plotting so that they could present the results of experiments in readily usable form. They were to be not routine workers but intelligent assistants. The research division had considered well the use to be made of them and had planned the content of the course with care.

The purpose of the training can best be explained by discussing one basic law of aerodynamics and one simple experiment which an aide might perform as a result of it. We know that we live at the bottom of a vast sea of air. A hand held palm upward is subject to air pressing on it from above. With equal pressure the air pushes on the back from below. The sides of the hand experience a like pressure. At every point the air is pressing in a direction at right angles to the surface of the skin and always with the same number of pounds. This pressure is called static pressure, for it is the force the air exerts merely by being there. If, however, a wind is blowing upon the palm of the hand, then there is a pressure on the palm that is not exerted on the back. This is called dynamic pressure, for it is caused by the moving air striking the hand. It is always exerted in the direction of the air stream and is proportional to the square of the air speed. A law of aerodynamics says that in steady flow the sum of static and dynamic pressures is constant; hence as dynamic pressure increases, static pressure decreases.

Consider now an airplane wing moving through the air; or, what is aerodynamically equivalent, air flowing past a still wing. In Figure 1 let w represent a cross section of the wing. If the air flow is steady, then all the air that at one moment is a position ACF has at a later moment reached BEG. The air below and far above the wing moves in a relatively straight line. That which flows over the top of the wing, however, must take the long path CDE. Since the time of its travel is the same as that for AB, its speed must be greater. Its dynamic pressure is therefore increased; whence its static pressure is de-

* As a result of war experience many positions in engineering and industry will no doubt be available for students with the proper mathematical background. This should have an important bearing on the teaching of mathematics in high schools and colleges.—Editor.

creased. There is now less pressure acting at right angles to the top of the wing tending to push it down than is acting at right angles to the bottom of the wing tending to push it up. Hence the wing rises and carries the airplane with it. The force tending to push the plane upward is called, appropriately, the lift force, or lift. If it just equals the weight of the plane, there will be level flight. If it is greater than the weight, the plane will climb. If it is less than the weight, the plane will descend.

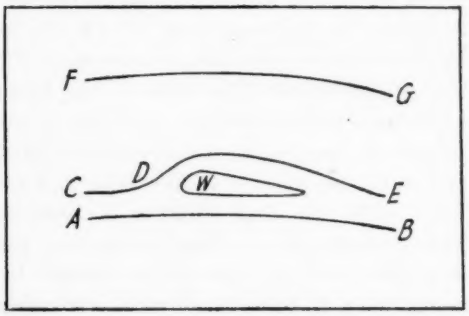

FIG. 1

When the pilot wants more lift, he must further increase the velocity of the air over the top of the wing. One way of doing this is to fly faster, but greater speed is not always possible or desirable. The only other way is to lengthen the path of the air. The pilot cannot change the shape of the wing, for it is rigid. He can, however, tilt up the airplane so that the angle of attack between the wing and the direction of the air is increased. Then the streamline *CDE* is more violently deflected, has a longer path, higher velocity, and consequently decreased static pressure. As the angle of attack increases, the lift increases until a critical position is reached. Here the air no longer flows smoothly around the upper surface, but instead breaks away in eddies and vortices. The lift drops off sharply. Sometimes this break-away occurs simultaneously all across the span of the wing. The pilot then has no warning until suddenly his plane has lost altitude. Sometimes the stall begins at one point and spreads rapidly. In this case the pilot feels the plane getting out from under control and knows to quickly decrease the angle of attack. It follows that a wing must be constructed not only to give sufficient lift but also to have as favorable a condition of air break-away as possible.

Let us suppose that an engineer has designed a new wing and that he wants a small model of it tested in a small wind tunnel. He sends drawings of the model to the shop; then tells an aide to run a lift experiment at a certain Reynolds Number. This number is the product of a constant times the wing span times the air speed. It permits calculation of the desired wind velocity; since the constant for the tunnel is known, and the blueprint gives the wing span. When the model comes from the shop, the aide checks it against the drawings. If it is satisfactory she mounts it on three struts rising from the floor of the test chamber of the wind tunnel. The two front struts are fixed, but the rear one can be raised or lowered to change the angle of attack. When she turns on the wind, the wing tries to rise and carry the floor of the test chamber with it. This floor is constructed somewhat like the platform of a balance. On a dial the aide reads the number of pounds pressure needed to keep the floor in its original position and records it as the lift. She turns off the air, changes the angle of attack, and repeats the procedure.

Let us suppose that at a relatively small angle the lift drops off. The aide is surprised, but she goes on with the experiment. If thereafter the lift fluctuates, she knows that something is wrong. It may be with the mounting of the wing, or with the air stream, or with the connections to the balance dial. Her acquaintance with the theory of lift is sufficient to convince her that results are erratic and that further experimentation would be a waste of time. She seeks the trouble. If she can fix it, she does; if not, she asks for help. Let us suppose that now all is well and she finishes her taking of data.

The engineer means to compare the performance of this wing with that of others developed by his colleagues or himself. Models of these were probably not constructed with uniform wing span; hence experiments with them will have been run at varying air speeds. The actual numbers of pounds lift for a given angle of attack are therefore not comparable. Quantities dependent only on the Reynolds Number must be used. These are the constants of variation in the formula expressing the fact that for a given angle of attack lift is proportional to the dynamic pressure and to the wing surface. The aide reckons the dynamic pressure, reads the wing surface from the blueprint, and calculates these coefficients of lift. She plots them against angle of attack and draws a smooth curve showing their trend. Let us hope that it rises steeply and is well rounded at the top, for then the wing gives good lift and the break-away of the air comes only gradually.

The engineer is interested in many properties of this wing beside the lift. During the progress of the experiment other aides have been taking other measurements. Each is to plot a chart of her data. When the engineer looks at the assembled material, he wants the information in it to stand out. He does not want to be distracted by various types of printing and of forming the numbers used along the axes. Variety is desirable in art but not in statistical work. The aides have therefore been taught that titles are to be three spaces high, other words two spaces. Lettering is upper case except for subscripts and superscripts. All letters have a slope of two to one, and each has special rules for width, etc. When the aides have finished, the report looks as though only one person had been working on it. The engineer will find immediately the information he seeks without being distracted by the descriptive matter.

It is clear that aides must know what they are doing, and why, in even this simple experiment. Their understanding is increasingly important as the work becomes more complex, such as that with propellers considered as thin twisted wings whose lift is the thrust which carries the airplane forward, or that in the field of thermodynamics on engines and jet propulsion.

In order to cover the desired amount of theory and practice in the training course, classes met for eight hours a day, five days a week. The continued presence in class of the aides made it possible to keep each one working at her proper speed. The diversity of subject matter permitted change from theory to technique and vice versa when signs of fatigue appeared. Ordinarily the daily routine proceeded as follows. The first two hours, while minds were fresh and alert, were spent on theory. About ten o'clock, when the sun was high enough to eliminate glare from the room, curve plotting or slide rule calculation was done. After an hour's break for lunch, the close eye work was continued. The last two hours were spent on problems or on easier theory, and frequently included time at the blackboard.

Twice a week the Research Division sent down one of its engineers to lecture on his specialty. These men did an outstanding job of clearly explaining the fundamentals of their fields without too much simplification of theory. Moreover, so great was their confidence in the aides and their interest in their work, that their enthusiasm was contagious. The aides felt from the beginning that they were becoming an important part in an important work. Approximately a third of our time was devoted to preparing for, hearing, quizzing on, and working problems suggested by these lectures. Their subjects included mechanics and strength of materials; a group on elementary aerodynamics and theory of flight; another group on gas laws, standard atmosphere, pressure and temperature measurements, and engines and engine testing; a set on propulsion; and finally a discussion of wind tunnels and their operation.

A quarter of our time was spent on let-

tering and plotting. In addition to the rules already mentioned, conventions with regard to titles, captions, etc., were taught. The technique of handling French curves was developed. Only experience can teach the proper fairing of curves, especially when the plotted points are taken from experimental data. The aides were encouraged to search available literature to find charts of the desired type, in order that they might benefit not only by the manner of drawing but also by the accompanying discussion. Student work was checked with a magnifying glass, for generous use of red pencil was believed to reinforce insistence on good form and accuracy which had to be learned in so brief a session.

The remaining time was divided equally between engineering drawing, taught by a member of the art department, and slide rule work. It was necessary to use the Log Log Duplex Trig slide rules; for formulas included trigonometric, exponential, and logarithmic functions.

At the conclusion of the course the aides were placed so far as possible according to their preferences and aptitudes. Their jobs range from running experiments as described above to abstracting articles in technical journals, and from making theoretical calculations to donning coveralls and working with engines. Their preparation was exacting and strenuous, but as a result of it they have handled their assignments with competence, understanding, satisfaction, and enjoyment.

Business arrangements admit the aides to the employment of the Corporation at the beginning of the course. Their expenses at the college are paid, and they are given in addition a small personal allowance. The corporation furnishes all materials used: plotting paper, drawing instruments, slide rules, reference books, as well as many ingenious models for purposes of illustration.

Administratively the course is a part of the summer session offering of the Department of Mathematics and is open to all qualified persons. Actually none but the aides have ever enrolled. The Corporation pays tuition, room, and board for them to the college. Instructors' salaries come from the summer school budget. Reports are made to the registrar's office even though the course carries no college credit.

An adaptation of the theoretical part of this training is offered as a regular three-point college course during the first term of the summer session. Academic prerequisites for admission are more stringent than those for the training, hence the students should master the material in ordinary class time plus outside preparation. They attend the corporation lectures thus learning principle and application from men engaged in research with both. They do interesting work in mathematics and theoretical physics while they learn how and why an airplane flies.

The training course has therefore provided the corporation with aides carefully prepared for its work, has introduced these young women to an interesting new vocation, and has enriched the curriculum of a liberal arts college.

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Field Project in Junior High School Mathematics

By GLADYS PYATT
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THE IDEA is rapidly gaining recognition that elementary mathematics would profit greatly from the introduction of field and laboratory work. Arithmetic has too often been taught as a skill unrelated to life outside the classroom. If arithmetic is to be fully meaningful, greater care must be taken to assure understandings that function in daily life. In this paper is presented a unit of work that was carried out with pupils on the eighth grade level in which they were taken out of the classroom for observation and first hand information.

AIMS

The general objectives of the project were to add interest and appreciation in the work in mathematics by demonstrating simple applications of mathematics to practical field problems, and to make these applications clear and meaningful to the students. The specific aims were:

1. To develop the ability to compute heights of objects by the shadow method and also by use of the sextant.
2. To develop the ability to estimate distances with a reasonable degree of accuracy.
3. To learn how to make scale drawings.
4. To become acquainted with the reading of electric meters.
5. To learn from lumber dealer the mathematics in the lumber business needed by the ordinary citizen.
6. To develop an understanding of the relationship between the circumference and diameter of a circle.

ACTUAL EXPERIENCES

Measuring Heights of Objects: The first object chosen for this activity was a small tree in order that actual measurement could be made so the students might verify their measuring and computation. Before going into the school yard for the field work, the pupils were assigned their particular tasks,—measuring the shadow of the tree, measuring the height of the pole, measuring the shadow of the pole, and recording of data. For each object measured, different students were assigned to the measuring tasks. Other objects used in the activity were a flag pole, a telephone pole, and a larger tree. After all necessary measurements in the field were made, the students returned to the classroom to do the computation. This work was done individually, after which results were compared, errors noted, and corrections made.

Approximate heights of objects were also found by use of an inexpensive sextant and a table which contained the degrees for angles of elevation with approximate trigonometric ratios for the tangents of the angles.

Estimating Distances: Estimations of the length, width, and height of the school room were made by each pupil in the room. Students were assigned the task of taking the actual measurements. These were taken after each estimation and comparison made in order that students might profit in their next estimation. The above procedure was also carried out in estimating the size of the hall. Distances outside the building were estimated and then measured,—the length and height of the school building, the court between two parts of the building, and the length of the sidewalk from the school to Main Street. Scale drawings were made by the students. Each student chose his own scale to be used in the drawing.

Reading Meters: Before going to read electric meters, the students were shown pictures of meters and an explanation was given as to how they were read. The pupils were taken to a nearby apartment house where a number of meters were available. Each student was given an opportunity to read one of the meters. After returning to the classroom the students were shown

how light bills were computed and also given the opportunity to figure bills using the rates of the local light company.

Visiting Lumber Yard: The students made the visit to a nearby lumber yard. They were given, through explanation and observation, the terms which the lumber merchant felt the average citizen should know. He explained to the students how computation is made when selling lumber and also which units were used in selling the various building products. Upon returning to the school the information given the students was discussed by the class.

Measuring Pole: Certain students were appointed to measure the circumference of the pole and record data, while the remainder of the class observed the measuring. Each member of the class then computed the length of the diameter using the measurement recorded.

Results

The following skills were acquired:

1. Ability to compute heights of objects by the shadow method.
2. Ability to use the sextant.
3. Ability to do scale drawing.
4. Ability to read electric meters.
5. Ability to estimate distances with reasonable accuracy.

Another outcome of the project was an enriched vocabulary (board feet, sextant, ratio, proportion, degree, angle of elevation, meter, kilowatt hour, scale drawing, accuracy, estimation, pi, circumference, and diameter).

Habits and attitudes acquired were:

1. Increased interest and appreciation in mathematics.
2. Desire to do mental arithmetic with accuracy as was demonstrated by the lumber merchant.
3. Desirable citizenship habits,—self control, cooperation, and respect for ideas of others.
4. Desirable work habits,—neatness, accuracy, checking work, and ability to follow directions.

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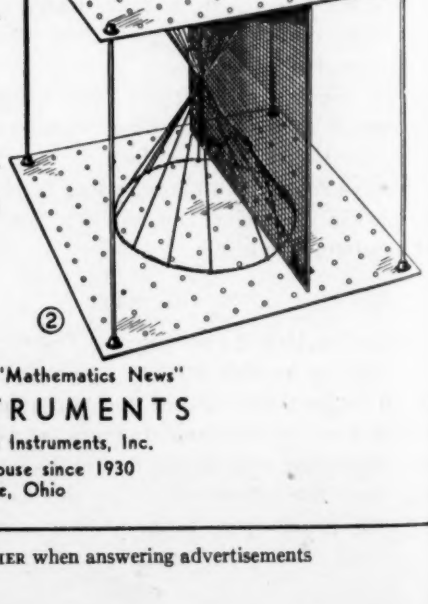

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Practical Mathematics

By HAROLD E. BOWIE
American International College, Springfield, Mass.

WORLD WAR II has focused our attention on the practical side of mathematics. Training in this subject has been a necessary part of the programs for turning out fliers, engineers, teachers, doctors, chemists, meteorologists and other workers. The amount of mathematics needed varies with the type of work and the level upon which the individual is eventually to perform. A test chemist may find use for only a little arithmetic and proportion. On the other hand, a research chemist may be handicapped without some graduate courses.

These fields for mathematically trained people existed before the war. The number of persons needed for such purposes was small compared to present requirements. The existence of a peace-time nucleus was a fortunate condition as it gave us something upon which to build. Our American International College graduates in mathematics of past years helped to lay the foundation.

New inventions and scientific discoveries have expanded the domain of mathematical applications. Much of this will be useful after the conflict is over. Emphasis upon mathematics in the preparatory schools has naturally increased. It is to be hoped that this influence will carry over to some extent in peace time. The fact that many secondary school pupils do not study any mathematics after the tenth grade has been a bad feature of our educational system. Coupled with further postponement by many in college, this accounts for some of the failures in the subject.

The accelerated programs are indispensable at the present time, as many trained persons are needed quickly. Within a few months the student is exposed to the important topics of undergraduate mathematics and may be handling ideas which are ordinarily reserved for graduate study. This policy can only be defended on the grounds of emergency requirements. Deep understanding and rigor must be sacrificed because of lack of time and mathematical maturity. Evidently this sort of superficial training can well be discontinued after the war.

Educators will derive some benefits from experience with war courses. They may find some of the new materials and procedures better than the old ones. We may wish to retain some of these after subjecting them to trial and criticism in normal times. The services of A.I.C. to the war effort have not made it necessary to introduce drastic changes in our courses. The acceleration of our program has not involved cutting down on clock-hours or course content. It is well that some institutions like our own are turning out at least a few persons with complete and thorough backgrounds in undergraduate mathematics.

After the war, the emphasis on the practical side of mathematics in colleges will gradually decrease to quite an extent. Practical mathematics will still be important but not to nearly as many people. Those who like mathematics for its own sake will help to prevent it from being submerged too greatly. Just as the best musicians, painters and writers are those who place their work first and the mercenary motive in second place, the best mathematicians are often those who are not excessively preoccupied with finding uses for the subject. Too much concern about its applications fetters the mind in its search for truth.

There should be little concern, however, about any conflict between the practical and cultural phases of the study of mathematics. The discoveries of those interested in pure mathematics may become practical

at any time as history has shown. Many of the developments of science would have been unlikely but for the ground work in mathematics laid by the theoretical workers. It is better for the person who loves pure mathematics to leave much of the attention to applications for the mathematical physicist or other applied scientist. An inventor may turn out many new ideas which are never used for one reason or another. Likewise, the mathematicians are continually developing new bits of theory which may or may not fit immediately into the science picture.

Undergraduate college mathematics can all be included in the practical category. It is comparatively elementary and is only the ABC's for the mathematics used in science and advanced mathematics, both applied and theoretical. On account of this, we can classify most college teachers and students of mathematics as being in the practical and not the theoretical field.

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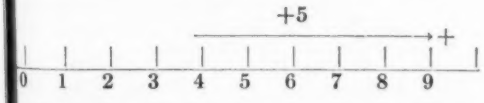
◆ THE ART OF TEACHING ◆

An Alternative Technique for Teaching Subtraction of Signed Numbers

By LAWRENCE M. HENDERSON
Wethersfield High School, Kewanee, Ill.

THE TECHNIQUE as outlined by W. E. Romig in the January MATHEMATICS TEACHER for teaching subtraction of signed numbers is a graphical representation of the mechanical process of changing the sign of the subtrahend and proceeding as in addition. I believe that pupils will find the following method more meaningful; my pupils prefer it to the mechanical method which they find outlined in our text. Too, many teachers who use this plan never introduce the mechanical one at all.

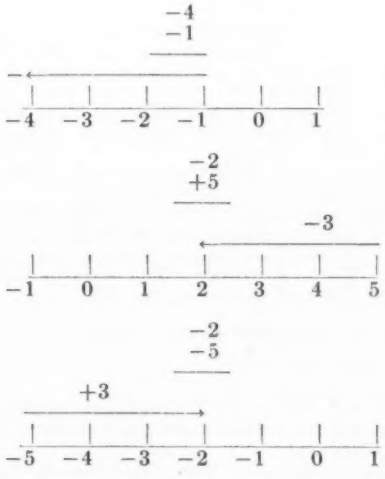
In teaching subtraction of signed numbers, I first draw a number scale. Zero is shown to be the division point between positive and negative numbers. There are only two directions in which one may go upon this scale: the positive direction is left to right from any point on the scale; the negative direction is right to left from any point on the number scale. Let us first study an example of subtraction in arithmetic: Subtract 4 from 9.



Start at 4 and count the units between 4 and 9. In arithmetic the direction is always from left to right, so it is logical to picture as negative the direction from right to left. The above problem should be followed up by using 9 as the subtrahend and 4 as the minuend. The absolute value remains the same, but the sign is negative. In other words, when we subtract two quantities, the absolute value of the difference equals the number of units which separate the subtrahend from the minuend, the sign being determined by the direction traveled in going from the subtrahend to the minuend.

In algebraic subtraction, irrespective of the absolute value of the subtrahend, the difference is always positive if the subtrahend is smaller than the minuend; the remainder is negative whenever the subtrahend is greater than the minuend. This can easily be illustrated by number scales.

Examples similar to the following should be included in the first discussion:



◆ IN OTHER PERIODICALS ◆

By NATHAN LAZAR

Midwood High School, Brooklyn 10, N. Y.

The American Mathematical Monthly

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1. Serge, B., "A Four-Dimensional Analogue of Pascal's Theorem for Conics," pp. 119–131.
2. Northrop, E. P., "Mathematics in a Liberal Education," pp. 132–137.
3. Mays, W. J. "The Valuation of Risks," pp. 138–148.
4. Bradley, A. D., "Azimuthal Equidistant Projection of the Sphere," pp. 148–151.
5. Ryser, Herbert, "Variations of Vector Inequalities," pp. 152.
6. War Information: New Selective Service Regulation; Notes on the Navy V-12 Program, pp. 171–173.
7. Reid, W. T. "Activities of the Committee on Examinations," pp. 173–176.

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1. Doyle, T. C., "Euclidean Metric Invariants of Conics by Tensor Algebra," pp. 179–187.
2. Kaplansky, Irving, "Lucas's Tests for Mersenne Numbers,' pp. 188–190.
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6. Discussions and Notes.
   (a) Farnell, A. B., "On the Second Derivative," p. 207.
   (b) Eves, Howard, "Checking the S. A. S. Case in Trigonometry," pp. 208–209.
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7. Frame, J. S., "Remarks on a Variation of Newton's Method," pp. 212–214.
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1. Editorial, pp. 1–2.
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3. Austin, D. J., "The Teaching of Logarithms," pp. 7–10.
4. Bailey, V. A., and Smith-White, W. B., "The Theory of Numbers or Interesting Arithmetic," pp. 11–15.
5. Turner, I. S., "Sir Percy Nunn," p. 15.
6. Mathematical Notes, pp. 16–22.
   (a) Hallet, P. H., "The Setting Out of Work for the Solution of Triangles."
   (b) Haron, T. K., "The Theorem of Three Perpendiculars."
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   (d) Oslington, C., "Note on the Graphing of Parabolas."
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   (f) Aitkin, A. G., "A Neglected Property of the Isosceles Triangle."
7. Book Review, p. 23.
8. Gillings, R. J., "Perpetual World Calendar."
9. Problem Bureau.
10. News and Notices.

The Duodecimal Bulletin

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1. Andrews, F. Emerson, "New Year Greetings," p. 1.
2. Smith, F. Morton, "John Benkow," pp. 3–4.
3. Terry, George S., "On Constructing a Table of Consecutive Squares," p. 9.
4. Camp, Kingsland, "On Multiplication Tables," p. 10.
5. Miscellaneous: Salutation; Endowment; Annual Meeting, New Friends, Bibliography; Res Publicae; Arithmetrix; Mathematical Cryptogram; The Special Committees; Mo for Megro; The Duodecimal Society of America.

National Mathematics Magazine

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1. Sanders, S. T., "Mathematicians and Their Inspirations," p. 210.
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3. Baten, William Dowell, "Analyzing Degrees of Freedom into Comparisons When the 'Classes' Do not Contain the Same Number of Items," pp. 221–228.
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7. Zant, James H., "Mathematics As a Field of Specialization for College Students," pp. 247–253.
8. Farnsworth, Ray D., "Stumbling Upon Laws," pp. 254–255.

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3. DeCicco, John, "Equilong Geometry of Third Order Differential Elements," pp. 276–282.
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6. Boyer, Carl B., "Historical Stages in the Definition of Curves," pp. 294–310.
7. Ransom, William R., "Multipliers Instead of Ratios," (continued), p. 311.
8. Morley, Raymond K., "Successive Derivatives of tan x," pp. 311–312.

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5. Nyberg, Joseph A., "Notes from a Mathematics Classroom," pp. 443–447.
6. Wayne, Alan, "Some Light Trigonometry," pp. 453–456.
7. Carnahan, Walter H., "What is Going to Become of High School Mathematics?", pp. 463–469.
8. Rosenthal, Jerome, "Geometric Goldbricks," pp. 472–478.

OFFICIAL NOTICE BY EDWIN W. SCHREIBER, SECRETARY

The National Council of Teachers of Mathematics

As secretary of the National Council of Teachers of Mathematics, I officially announce the annual election of certain officers of the National Council, said election to take place February, 1946.

At the Atlantic City meeting, Feb. 26, 1938, the Nominating Committee, consisting of the two most recent ex-presidents and the secretary as chairman (for this year: Mary A. Potter, Rolland R. Smith, and Edwin W. Schreiber), was instructed to prepare an official ballot naming two eligible candidates for each elective office, reserving a blank space for a third prospective candidate whose name may be written in by the voter. The officers to be elected are: President 1946–1947, Second Vice-President 1946–1947, and three Directors, 1946–1948. The official ballot will appear in the February issue of the MATHEMATICS TEACHER, 1946.

The periods of service of the officers of the National Council, from its organization in February, 1920 to the present time are printed on the following page.

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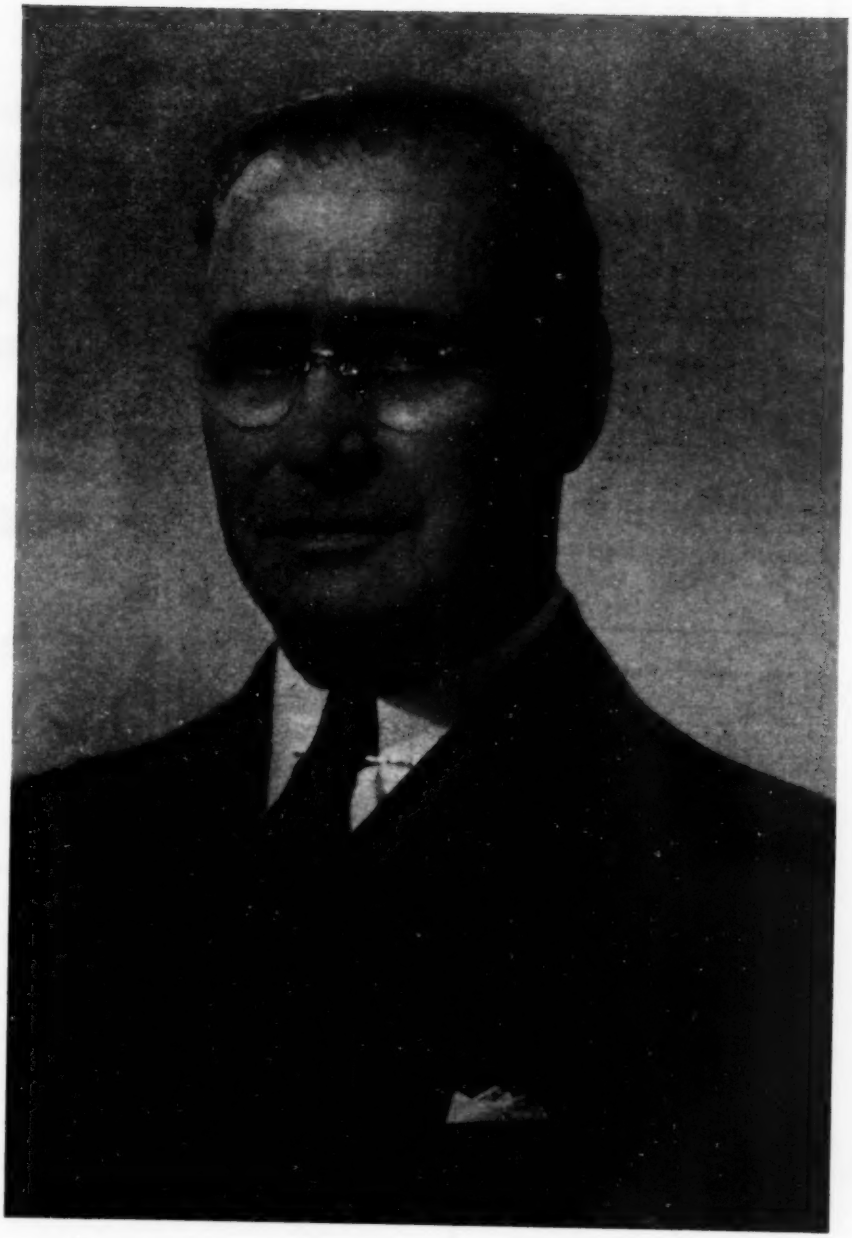
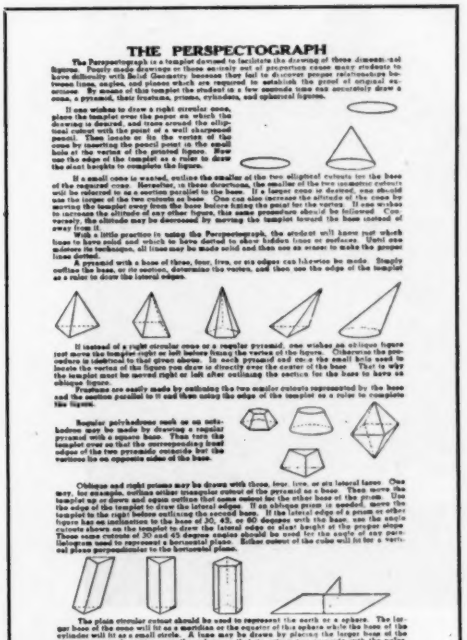


C. M. AUSTIN
First President of The National Council of Teachers of Mathematics

To Teachers of Solid Geometry

The "PERSPECTOGRAPH" is a templet made of a good grade cardboard which will enable every student to accurately draw the three dimensional figures studied in solid geometry in just a few seconds time. Each templet is enclosed in an envelope on which detailed instructions of its use are printed.



THE PERSPECTOGRAPH

The Perspectograph is a templet devised to facilitate the drawing of three dimensional figures. Poorly made drawings or those actually out of proportion cause many students to have difficulty with Solid Geometry because they fail to discover proper relationships between lines, angles, and planes which are required to establish the proof of original exercises. By means of this templet the student in a few seconds time can accurately draw a cone, a pyramid, their frustums, prisms, cylinders, and spherical figures.

If one wishes to draw a right circular cone, place the templet over the paper on which the drawing is desired, and trace around the elliptical cutout with the point of a well sharpened pencil. Then locate or fix the vertex of the cone by inserting the pencil point in the small hole at the vertex of the printed figure. Now use the edge of the templet as a ruler to draw the slant heights to complete the figure.

If a small cone is wanted, outline the smaller of the two elliptical cutouts for the base of the required cone. Hereafter, in these directions, the smaller of the two isometric cutouts will be referred to as a section parallel to the base. If a larger cone is desired, one should use the larger of the two cutouts as base. One can also increase the altitude of the cone by moving the templet away from the base before fixing the point for the vertex. If one wishes to increase the altitude of any other figure, this same proceedure should be followed. Conversely, the altitude may be decreased by moving the templet toward the base instead of away from it.

With a little practice in using the Perspectograph, the student will know just which lines to have solid and which to have dotted to show hidden lines or surfaces. Until one achieves its technique, all lines may be made solid and then use an eraser to make the proper lines dotted.

A pyramid with a base of three, four, five, or six edges can likewise be made. Simply outline the base, or its section, determine the vertex, and then use the edge of the templet as a ruler to draw the lateral edges.

If instead of a right circular cone or a regular pyramid, one wishes an oblique figure just move the templet right or left before fixing the vertex of the figure. Otherwise the procedure is identical to that given above. In each pyramid and cone the small hole used to locate the vertex of the figure can draw as directly over the center of the base. That is why the templet must be moved right or left after outlining the section for the base to have an oblique figure.

Frustums are easily made by outlining the two similar cutouts represented by the base and the section parallel to it and then using the edge of the templet as a ruler to complete the figure.

Regular polyhedrons such as an octahedron may be made by drawing a regular pyramid with a square base. Then turn the templet over so that the corresponding bent edges of the two pyramids coincide but the vertices be on opposite sides of the base.

Oblique and right prisms may be drawn with three, four, five, or six lateral faces. One may, for example, outline either triangular cutout of the pyramid as a base. Then move the templet up or down and again outline that same cutout for the other base of the prism. Use the edge of the templet to draw the lateral edges. If an oblique prism is needed, move the templet to the right before outlining the second base. If the lateral edge of a prism or other figure has an inclination to the base of 30, 45, or 60 degrees with the base, use the angle cutouts shown on the templet to draw the lateral edge or slant height at the proper slope. These same cutouts of 30 and 45 degree angles should be used for the angle of any parallelogram used to represent a horizontal plane. Either cutout of the cube will fit for a vertical plane perpendicular to the horizontal plane.

The plain circular cutout should be used to represent the earth or a sphere. The larger base of the cone will fit as a meridian or the equator of the sphere while the base of the cylinder will fit as a small circle. A base may be drawn by placing the larger base of the cone in a vertical position over the sphere so that its extremities coincide with the poles. If the whole figure is drawn with a solid line, the whole figure will appear on the same side of the sphere as in the case of the base. If half is solid and half dotted, it will be a meridian. Spherical and polar triangles are included on the templet because they are so difficult for the student to draw.

For the present the Perspectograph is made of cardboard but later will be made of celluloid or a transparent plastic. At that time a set of templets with figures similar to those on the small templet, only much larger, will be available for blackboard use.

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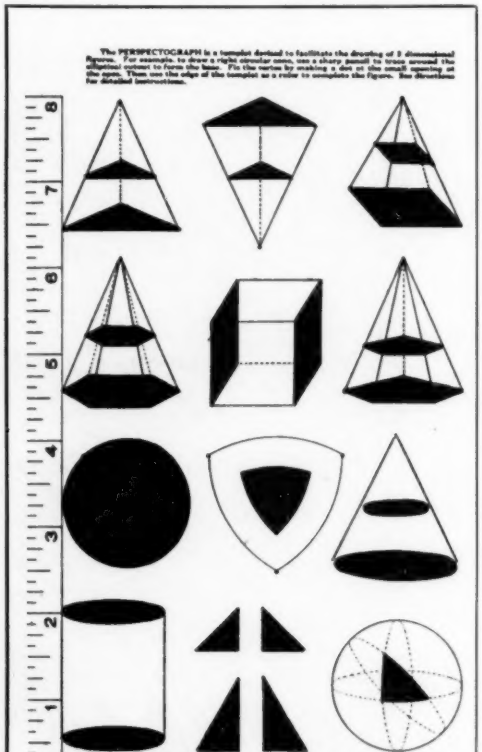

The PERSPECTOGRAPH is a templet devised to facilitate the drawing of 3 dimensional figures. For example, to draw a right circular cone, use a sharp pencil to trace around the elliptical cutout to form the base. Fix the vertex by making a dot at the small opening at the apex. Then use the edge of the templet as a ruler to complete the figure. See directions for detailed instructions.

MARGARET JOSEPH, Milwaukee 2, Wisconsin

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